Verifying Trigonometric Identities

Steps for Verifying Trigonometric Identities

1. Brainstorm!
   - Write down any potentially useful theorems, identities, etc.
   - Decide which side of the equation you want to rewrite (usually it's easier to simplify the more complicated side).
2. Can you rewrite the expression with what you brainstormed?
   - If not:
     - Can you rewrite the expression in terms of sine and cosine?
     - Did you miss any trigonometric identities (Pythagorean identities, reciprocal identities, etc.) you can substitute in? Double check!
     - Could you rearrange or rewrite the expression so you can substitute in a trigonometric identity?
     - Are there any expressions you can factor or expand?
     - Could you multiply by a conjugate?
     - Is there an expression equal to 1 that you can multiply by? (ex. $\frac{\sin\theta}{\sin\theta}$)
   - If you can, great work! Once you've rewritten one side of the equation to match the other, you've verified the identity!
3. Stuck? Try a different direction or a different way of verifying the identity. Keep in mind that there are multiple ways of verifying a trigonometric identity.

Example 1:

Verify the following trigonometric identity:

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\csc\theta$$

We will simplify the left side of the equation, since it is more complicated. Since we want our final product to be a single trig function, let's find a common denominator and combine the quotients.

$$\frac{\sin^2\theta}{\sin\theta\,(1+\cos\theta)}+\frac{(1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}\Rightarrow\frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}$$

Next, we can multiply out the $(1+\cos\theta)^2$

$$\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta\,(1+\cos\theta)}$$

We can rearrange the numerator, so we have

$$\frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta\,(1 + \cos\theta)}$$

Substituting the Pythagorean Identity ($\sin^2\theta + \cos^2\theta = 1$), we get

$$\frac{2 + 2\cos\theta}{\sin\theta\,(1 + \cos\theta)}$$

Factoring the numerator,

$$\frac{2(1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)}$$

Simplifying

$$\frac{2}{\sin\theta}$$

Using the reciprocal identity $\left(\frac{1}{\sin\theta} = \csc\theta\right)$,

$$2\csc\theta$$