Solving Systems of Linear Equations

Method of Substitution:

Solve the given system of equations:

$$\begin{cases} 2x + 3y = 3 \\ 3x - 4y = 13 \end{cases}$$

STEP ONE: Number each equation.

$$(1)\ 2x + 3y = 3$$

$$(2)\ 3x - 4y = 13$$

STEP TWO: Choose one equation and solve for one variable.

$$(1)\ 2x + 3y = 3$$

$$2x = 3 - 3y$$

$$\boldsymbol{x = \frac{3}{2} - \frac{3}{2}y}$$

STEP THREE: Substitute the expression in step two for the variable in the second equation and solve for the remaining variable.

$$(2)\ 3\left(\frac{3}{2} - \frac{3}{2}y\right) - 4y = 13$$

$$\frac{9}{2} - \frac{9}{2}y - 4y = 13$$

$$9 - 9y - 8y = 26$$

$$9 - 17y = 26$$

$$-17y = 17$$

$$\boldsymbol{y = -1}$$

STEP FOUR: Substitute the solution for the variable you solved for into either of the original two equations and solve for the other variable.

$$(1)\ 2x + 3(-1) = 3$$

$$2x - 3 = 3$$

$$2x = 6$$

$$\boldsymbol{x = 3}$$

STEP FIVE: Write your solution as an ordered pair.

$$\boldsymbol{(3, -1)}$$

Method of Elimination:

Solve the given system of equations:

$$\begin{cases} 2x + y + 2z = 11 \\ 3x + 2y + 2z = 8 \\ x + 4y + 3z = 0 \end{cases}$$

STEP ONE: Number each equation.

$$(1)\ 2x + y + 2z = 11$$

$$(2)\ 3x + 2y + 2z = 8$$

$$(3)\ x + 4y + 3z = 0$$

STEP TWO: Choose any two equations and eliminate one variable using the elimination method. We will eliminate the y variable by multiplying equation (1) by -2 and adding it to equation (2). The new equation is equation (4).

$$\text{-}2(1) - 4x - 2y - 4z = -22$$

$$\underline{(2) \quad 3x + 2y + 2z = 8\ }$$

$$(4) \qquad -x - 2z = -14$$

STEP THREE: Choose a different pair of equations and eliminate the same variable. We will eliminate the y variable by multiplying equation (2) by -2 and adding it to equation (3). The new equation is equation (5).

$$-2(2) - 6x - 4y - 4z = -16$$

$$\underline{(3) \qquad x + 4y + 3z = 0}$$

$$(5) \quad -5x - z = -16$$

STEP FOUR: Equations (4) and (5) create a system of two equations in two variables. Eliminate one of the variables solve for the remaining variable, then substitute it back into equation (4) or

(5) to solve for the other variable in that system. We will multiply equation (5) by -2 to eliminate the z variable.

$$(4)\ -x-2z=-14$$

$$\underline{-2(5)\ 10x+2z=32\qquad}$$

$$9x=18$$

$$\boldsymbol{x=2}$$

$$(4)\ -(2)-2z=-14$$

$$-2z=-12$$

$$\boldsymbol{z=6}$$

STEP FIVE: Substitute the solutions for the two variables you have solved for into any one of the original three equations and solve for the third variable.

$$(3)\ (2)+4y+3(6)=0$$

$$2+4y+18=0$$

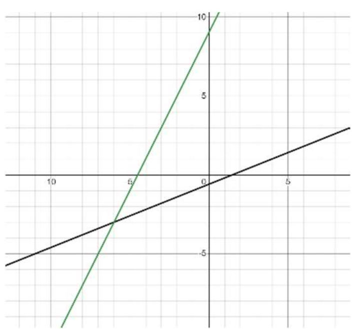
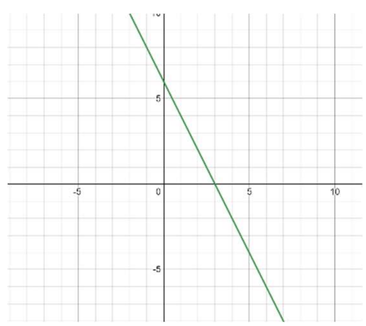
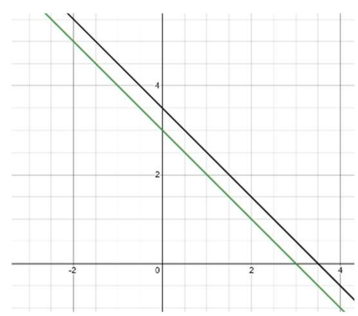
$$4y=-20$$

$$\boldsymbol{y=-5}$$

STEP SIX: Write your solution as an ordered triple.

$$\boldsymbol{(2,-5,6)}$$

Systems of Linear Equations in Two Variables:

Given	$\begin{cases} 2x - 5y = 3 \\ y - 2x = 9 \end{cases}$	$\begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases}$	$\begin{cases} x + y = 3 \\ 2x + 2y = 7 \end{cases}$
Solve Algebraically	$y - 2x = 9$ $\boldsymbol{y = 2x + 9}$ $2x - 5(2x + 9) = 3$ $2x - 10x - 45 = 3$ $-8x = 48$ $\boldsymbol{x = -6}$ $y - 2(-6) = 9$ $y + 12 = 9$ $\boldsymbol{y = -3}$ $\boldsymbol{(-6, -3)}$	$4x + 2y = 12$ $2(-2x - y = -6)$ $4x + 2y = 12$ $\underline{-4x - 2y = -12}$ $\mathbf{0 = 0}$ Since $0 = 0$ is a **true** statement, these equations will have the same slope and the same y-intercept. They are the same line.	$x + y = 3$ $\boldsymbol{y = 3 - x}$ $2x + 2(3 - x) = 7$ $2x + 6 - 2x = 7$ $\mathbf{6 = 7}$ Since $6 = 7$ is a **false** statement, these equations will have the same slope and different y-intercepts. They are parallel lines.
Solve Graphically	$\begin{cases} 2x - 5y = 3 \\ y - 2x = 9 \end{cases} \rightarrow \begin{cases} y = \frac{2}{5}x - \frac{3}{5} \\ y = 2x + 9 \end{cases}$ 	$\begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases} \rightarrow \begin{cases} y = -2x + 6 \\ y = -2x + 6 \end{cases}$ 	$\begin{cases} x + y = 3 \\ 2x + 2y = 7 \end{cases} \rightarrow \begin{cases} y = -x + 3 \\ y = -x + \frac{7}{2} \end{cases}$ 
Then We Say…	Consistent System Independent Equations One Unique Solution	Consistent System Dependent Equations Infinitely Many Solutions	Inconsistent System No Solution