Quadratic Function (Explanation & Examples)

Quadratic Function

where a, b, and c are real numbers with $a \neq 0$, the **function of the form**:

$$f(x) = ax^2 + bx + c$$

Standard Form of a Quadratic Function

$$f(x) = a(x-h)^2 + k, a \neq 0$$

The graph of f is a parabola with **vertex** (h, k).

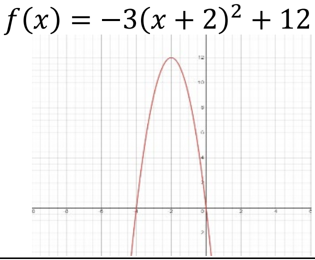
If $a > 0$, the parabola opens up, k is the **minimum value** of f ;
If $a > 0$, the parabola opens down, k is the **maximum value** of f .

Steps for finding the Vertex (h, k) **of** $f(x) = ax^2 + bx + c, a \neq 0$**:**

1. Find the **x-coordinate** $h = -\frac{b}{2a}$ of the vertex.
2. Calculate $k = f\left(-\frac{b}{2a}\right)$ to find its **y-coordinate**.
3. If $a > 0$, then k is the minimum value of f .
4. If $a > 0$, then k is the maximum value of f .

EXAMPLE

Sketch the graph of $f(x) = -3(x+2)^2 + 12$

Steps for graphing a Quadratic Function in Standard Form	$f(x) = -3(x+2)^2 + 12$
Step 1 The graph is a parabola because it has the form $f(x) = a(x-h)^2 + k$ Identify a, h, and k.	1. The graph of $f(x) = -3(x+2)^2 + 12$ $= -3[x - (-2)]^2 + 12$ ↑ ↑ ↑ a h k Is a parabola; $a = -3$, $h = -2$, and $k = 12$
Step 2 Determine how the parabola opens, if $a > 0$, the parabola opens up. If $a < 0$, it opens down.	2. Because $a = -3 < 0$, the parabola opens down.
Step 3 Find the vertex (h, k). If $a > 0$ or $(a < 0)$, the function f has a minimum (or a maximum) value k at $x = h$.	3. The vertex $(h, k) = (-2,12)$. Because the parabola opens down, the function f has a maximum value of 12 at $x = -2$.
Step 4 Find the x **- intercepts (if any).** Let $f(x) = 0$ and solve $ax^2 + bx + c = 0$. If the solutions are real numbers, they are the x - intercepts. If not, the parabola lies entirely above the x - axis (when $a > 0$) or entirely below the x - axis (when $a < 0$).	4. $0 = -3(x+2)^2 + 12$ Set $f(x) = 0$. $3(x+2)^2 = 12$ Add $3(x+2)^2$ to both sides. $(x+2)^2 = 4$ Divide both sides by 3. $x + 2 = \pm 2$ Square root property $x = -2 \pm 2$ Subtract 2 from both sides. $x = 0$ or $x = -4$ Solve for x . The x - intercepts are 0 and –4. The parabola passes through the points (0,0) and $(-4,0)$.
Step 5 Find the y **- intercept.** Replace x with 0. Then $f(0) = ah^2 + k$ is the y - intercept.	5. $f(0) = -3(0+2)^2 + 12 = 0$. The y - intercept is 0. As already shown, the parabola passes through the origin (0,0).
Step 6 Sketch the graph. Plot the points found in Steps 3-5 and join them to form a parabola. Show the axis $x = h$ of the parabola by drawing a sashed vertical line. If there are no x - intercepts, draw the half of the parabola that passes through the vertex and a second point, such as the y - intercept. Then use the axis of symmetry to draw the other half.	6. The axis of the parabola is the vertical line $x = -2$. The graph of the parabola is shown in the figure. $f(x) = -3(x+2)^2 + 12$ 

EXAMPLE

Sketch the graph of $f(x) = 2x^2 + 8x - 10$

Steps for graphing any quadratic function.	$f(x) = 2x^2 + 8x - 10$

Step 1 Identify $a, b,$ **and** c **.**	1. In the equation $y = f(x) = 2x^2 + 8x - 10$, $a = 2$, $b = 8$, and $c = -10$
Step 2 Determine how the parabola opens. If $a > 0$, the parabola opens up; If $a < 0$, the parabola opens down.	2. Because $a = 2 > 0$, the parabola opens up.
Step 3 Find the vertex (h, k)**.** Use the following formula: $$h = -\frac{b}{2a}$$ $$k = f\left(-\frac{b}{2a}\right)$$ k Is a minimum if $a > 0$. k Is a maximum if $a < 0$.	3. $h = -\frac{b}{2a} = -\frac{8}{2(2)} = -2$ $(a = 2, b = 8)$ $k = f(h) = f(-2)$ $= 2(-2)^2 + 8(-2) - 10$ Replace h with -2 $= -18$ Simplify. The vertex is $(-2, -18)$. The function f has a minimum value of -18 at $x = -2$.
Step 4 Find the x **- intercepts (if any).** Let $f(x) = 0$ and solve $ax^2 + bx + c = 0$. If the solutions are real numbers, they are the x - intercepts. If not, the parabola lies entirely above the x - axis (when $a > 0$) or entirely below the x - axis (when $a < 0$).	4. $2x^2 + 8x - 10 = 0$ Set $f(x) = 0$. $2(x^2 + 4x - 5) = 0$ Factor out 2. $2(x + 5)(x - 1) = 0$ Factor. $x + 5 = 0$ or $x - 1 = 0$ Zeros-product property. $x = -5$ or $x = 1$ Solve for x . The x - intercepts are at $x = -5$ and $x = 1$; the graph of f passes through the points $(-5,0)$ and $(1,0)$.
Step 5 Find the y **- intercept.** Let $x = 0$. The result, $f(0) = c$, is the y - intercept.	5. Set $x = 0$ to obtain $f(0) = 2(0)^2 + 8(0) - 10 = -10$The y - intercept is -10 and the graph passes through the point $(0, -10)$.
Step 6 The parabola is symmetric about its axis, $x = -\frac{b}{2a}$. Use this symmetry to find additional points.	6. The axis of symmetry is $x = -2$. The symmetric image of $(0, -10)$ about the axis $x = -2$ is $(-4, -10)$.
Step 7 Draw a parabola through the points found in Steps 3-6. If there are no x - intercepts, draw the half of the parabola that passes through the vertex and a second point, such as the y - intercept. Then use the axis of symmetry to draw the other half.	7. The parabola passing through the points in Steps 3-6 is sketched in the figure. $f(x) = 2x^2 + 8x - 10$