

# Logarithmic Properties

Property	Example
$y = \log_a x$	$y = \log_2 x$ or $y = \log_5 x$
$\log_a a^x = x$	$\log_2 2^x = x$ or $\log_2 2^5 = 5$
$a^{\log_a x} = x$	$2^{\log_2 x} = x$ or $5^{\log_5 2} = 2$
$\log_a 1 = 0$	$\log_2 1 = 0$ or $\log_5 1 = 0$
$\log_a a = 1$	$\log_2 2 = 1$ or $\log_5 5 = 1$
$\log x = \log_{10} x$	$\log 2 = \log_{10} 2$ or $\log 5 = \log_{10} 5$
$\ln x = \log_e x$	$\ln 2 = \log_e 2$ or $\ln 5 = \log_e 5$
$\log_a xy = \log_a x + \log_a y$	$\log_2 6 = \log_2 2 \cdot 3 = \log_2 2 + \log_2 3 = 1 + \log_2 3$ or $\log_5 10 = \log_5 5 \cdot 2 = \log_5 5 + \log_5 2 = 1 + \log_5 2$
$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\log_2 \frac{3}{4} = \log_2 3 - \log_2 4 = \log_2 3 - 2$ or $\log_5 \frac{1}{7} = \log_5 1 - \log_5 7 = 0 - \log_5 7 = -\log_5 7$
$\log_a x^b = b \log_a x$	$\log_2 5^3 = 3 \log_2 5$ or $\log_5 49 = \log_5 7^2 = 2 \log_5 7$
$\log_b x = \frac{\log_a x}{\log_a b}$	$\log_2 5 = \frac{\log_5 5}{\log_5 2} = \frac{\ln 5}{\ln 2}$ or $\log_4 8 = \frac{\log_2 8}{\log_2 4} = \frac{\log_2 2^3}{\log_2 2^2} = \frac{3 \log_2 2}{2 \log_2 2} = \frac{3}{2}$