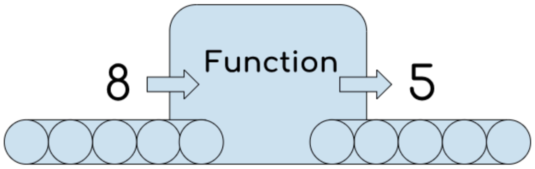
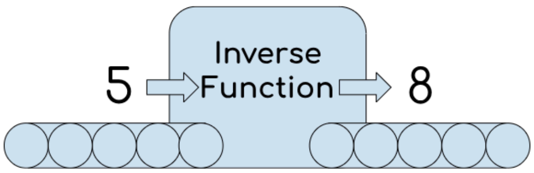
Inverse Functions:

The word inverse means “opposite” or “reverse”. So, if we start with a function that does something, its inverse is one that reverses it, or does the opposite.

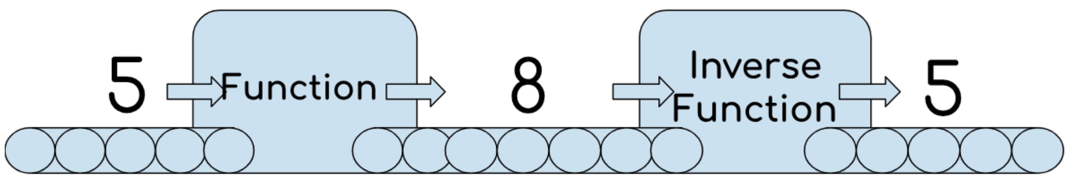


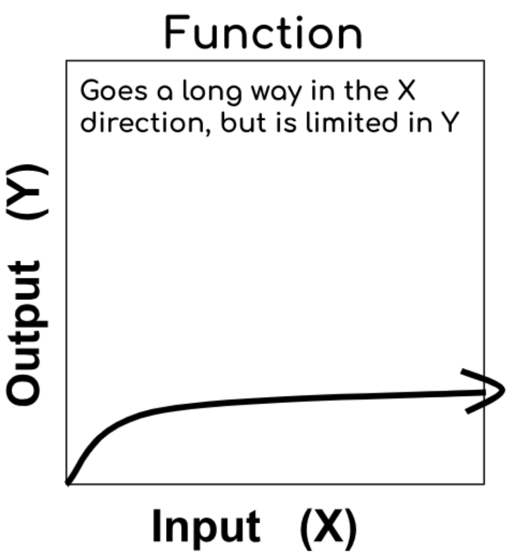
Let’s say that we have a function that takes in 8 as an input, and puts out 5.



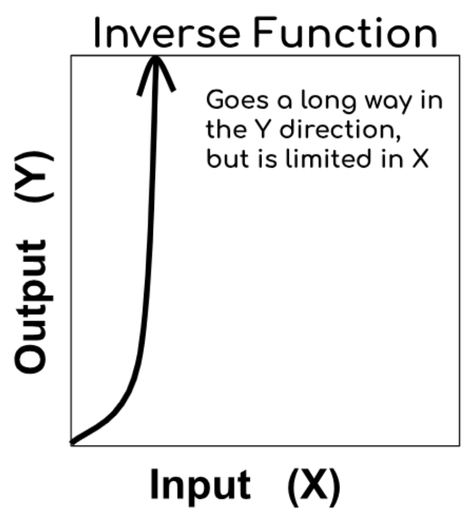
We want to find a function that does the reverse. In this case, if we gave the inverse function an input of 5, it would output 8.

If you put something through a function, and then its inverse, you get back what you started with. The original function does something to the input, and then the inverse undoes it.





If we graph the function, swapping the input and output looks like swapping X and Y. If the original function goes a long way in X, but doesn’t go far in Y, then the inverse will go a long way in Y, but not go far in X.



If you have a function written as an equation, you also find its inverse by switching X and Y, and then solving to get Y alone again.

Example 1:

$y = 15x - 30$

$\Downarrow$

$x = 15y = 30$

$\Downarrow$

$x + 30 = 15y$

$\Downarrow$

$\frac{x+30}{15} = y$

$\Downarrow$

$y = \frac{x+30}{15}$

Example 2:

$f(x) = x^3 + 7$

$\Downarrow$

$y = x^3 + 7$

$\Downarrow$

$x = y^3 + 7$

$\Downarrow$

$x - 7 = y^3$

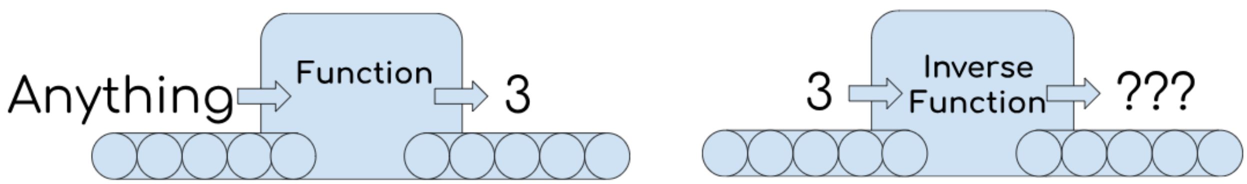
$\Downarrow$

$\sqrt[3]{x - 7} = y$

$\Downarrow$

$y = \sqrt[3]{x - 7}$

It’s important to know that not every function has an inverse. For example, if I have a function that outputs 3 no matter what you put in, there’s no way to undo it.



There is a nice visual way to know whether a function has an inverse function. If the function looks like something you could roll a ball down with no bumps, then it does have an inverse function.