# Infinite Series Convergence Tests

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| Limit Test/ Divergence Test | If $\lim_{n\to\infty} a_n = 0$, the series converges, otherwise it diverges |
| Ratio Test | If $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$, the series is absolutely convergent. If $> 1$, it diverges. If equal to 1, it is inconclusive. |
| Root Test | If $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$, the series is absolutely convergent. If $> 1$, it diverges. If equal to 1, it is inconclusive. |
| Integral Test | If $a_n$ is positive and decreasing, then let $f(n) = a_n$. Then if $\lim_{t\to\infty} \int_1^t f(x)dx < 0$, the series converges. If the integral diverges, the series diverges as well. |
| P-series Test | Let $k > 0$ . Then$\sum_{n=k}^{\infty}\left(\frac{1}{n^p}\right)$ converges if $p > 1$ . If n and k = 1, it is the Harmonic series, which diverges. |
| Direct Comparison Test | If $\sum_{n=1}^{\infty} b_n$ converges absolutely and $\lvert a_n\rvert \leq \lvert b_n\rvert$ for large enough $n$ , then $\sum_{n=1}^{\infty} a_n$ converges absolutely. |
| Limit Comparison Test | If series a and series b are both positive, and $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right)$ exists, is finite and nonzero, then either both series converge, or both diverge. |
| Absolute Convergence Test | If $\sum_{n=0}^{\infty}\lvert a_n\rvert = L$, where L is a real number, then the series converges absolutely. |
| Alternating Series Test | If:<br>$a_n$ are all positive,<br>$\lim_{n\to\infty} a_n = 0$, and<br>$a_n$ is always decreasing,<br>Then the series is convergent. |