Exponential & Logarithmic Functions

An **exponential function** is of the form

$$f(x) = b^x,$$

Where b is the base and $b > 0$ and $b \neq 1$.

A **logarithmic function** is of the form

$$f(x) = \log_b x,$$

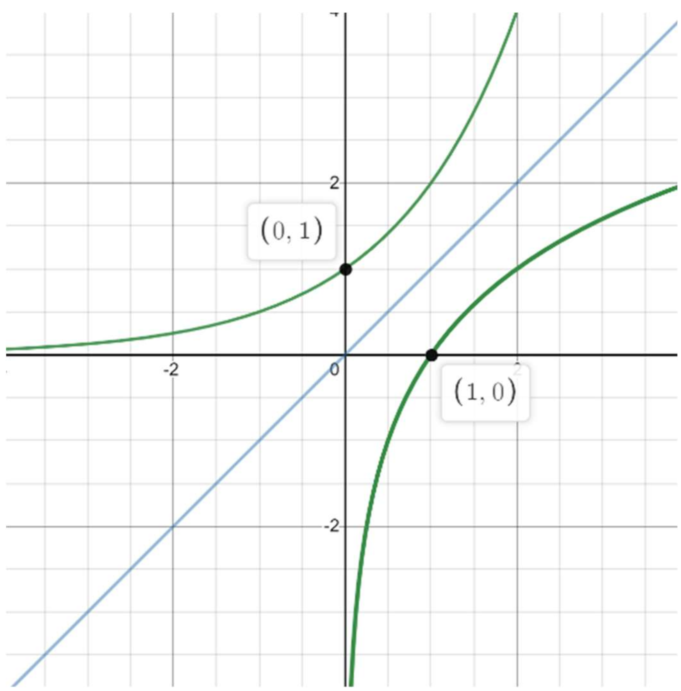
Where b is the base and $x > 0$, $b > 0$, and $b \neq 1$

Logarithmic and Exponential functions are **inverse functions**, which means that

$y = \log_b x$ is the same as saying $b^y = x$

and

$y = b^x$ is the same as saying $\log_b y = x$



Since Logarithmic and Exponential functions are inverse functions, their properties are related.

Properties of Exponents

$$b^0 = 1$$

$$b^1 = b$$

$$b^{-1} = \frac{1}{b}$$

$$b^x = b^x$$

$$b^m b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(b^m)^p = b^{m \cdot p}$$

Properties of Logarithms

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b \frac{1}{b} = -1$$

$$\log_b b^x = x \text{ for all } x, b^{\log_b x} = x \text{ when } x > 0$$

$$\log_b M \cdot N = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^p = p \cdot \log_b M$$

Special Logarithmic Functions

Common log: $\log_{10} x = \log x$

Natural log: $\log_e x = \ln x$

Change of base formula

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

Examples

$$2^3 = 8 \leftrightarrow \log_2 8 = 3$$

$$e^x = 5 \leftrightarrow \ln 5 = x$$

$$4 \log x = 2 \leftrightarrow \log x^4 = 2 \leftrightarrow 10^2 = x^4$$

Common Applications of Exponential Functions

Compound Interest

If $\$P$ is the initial deposit, and interest is paid n times per year at an annual rate of *r*, the amount A in the account after *t* years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Radioactive Decay

If A is the amount of radioactive material present at time *t*, A_0 was the amount present at $t = 0$, and *h* is the material's half-life, then

$$A = A_0 2^{-t/h}$$

Exponential Growth/Decay

If *P* is the population at some time *t*, P_0 is the initial population at $t = 0$, and *r* is the rate of growth/decay, then

$$P = P_0 e^{rt}$$

Common Applications of Logarithmic Functions

pH of a Solution

If [H+] is the hydrogen ion concentration in gram ions per liter, then

$$\text{pH} = -\log[H+]$$

Decibel Voltage Gain

If the output voltage to a device is E_0 volts and the input voltage is E_1, then the decibel dB gain is given by

$$\text{dB gain} = 20\log\frac{E_0}{E_1}$$

The Richter Scale

If *R* is the intensity of an earthquake on the Richter Scale, *A* is the amplitude (measured in micrometers) of the ground motion and *P* is the period (the time of one oscillation of the Earth's surface measured in seconds), then

$$R = \log\frac{A}{P}$$