

Completing the Square

Vertex form is a useful equation type to help graph a parabola. Completing the square is the method for writing a parabola equation in vertex form.

This equation is not very useful	$y = 2x^2 + 4x + 5$
Divide by coefficient of x^2	$\frac{y}{2} = x^2 + 2x + \frac{5}{2}$
$A = 1, b = 2, c = \frac{5}{2}$	$x^2 + 2x + \frac{5}{2}$
Take half of b and square it	$\left(\frac{1}{2} \cdot 2\right)^2 = (1)^2 = 1$
Add that number to both sides of the equation	$1 + \frac{y}{2} = x^2 + 2x + \frac{5}{2} + 1$
Rearrange right-hand side	$1 + \frac{y}{2} = (x^2 + 2x + 1) + \frac{5}{2}$
Parabola is a perfect square of $(x + 1)$ (1 is half of b)	$1 + \frac{y}{2} = (x + 1)^2 + \frac{5}{2}$
Subtract 1 from both sides	$\frac{y}{2} = (x + 1)^2 + \frac{5}{2} - 1$
Multiply by 2 (Continued on page 2)	$y = 2(x + 1)^2 + 5 - 2$
Simplify right-hand side, this is vertex form now	$y = 2(x + 1)^2 + 3$
So vertex of ours is $(-1, 3)$	$y = 2(x - (-1))^2 + 3$
Vertex formula:	$y = a(x - h)^2 + k$