

Basic Trigonometric Identities

Definitions of the Six Trigonometric Functions,
where θ is any angle:

$$\sin \theta = \frac{1}{\csc \theta} = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$$

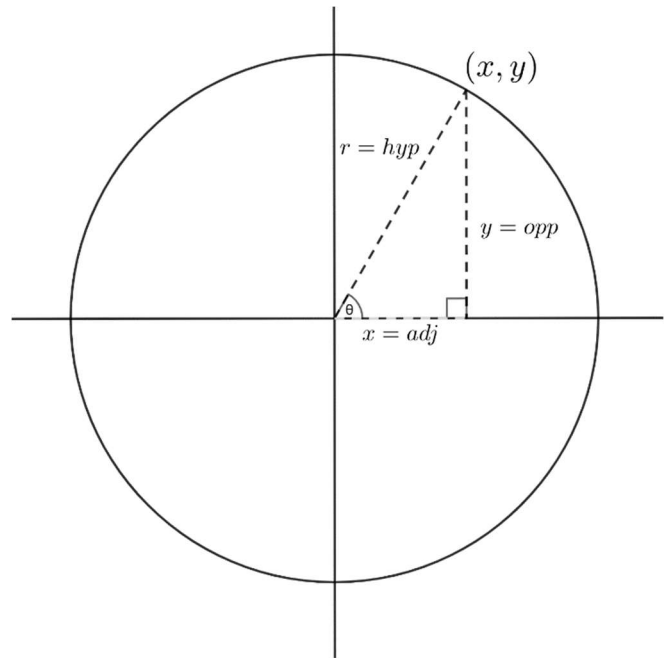
$$\cos \theta = \frac{1}{\sec \theta} = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{\cot \theta} = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

$$r = \sqrt{x^2 + y^2}$$



Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

By dividing by $\sin^2 \theta$ and $\cos^2 \theta$ respectively, we get two more Pythagorean identities:

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} \rightarrow 1 + \cot^2 \theta = \csc^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \rightarrow \tan^2 \theta + 1 = \sec^2 \theta$$

Even & Odd Trig Functions*:

$$\sin(-\theta) = -\sin \theta \text{ (odd)}$$

$$\cos(-\theta) = \cos \theta \text{ (even)}$$

Co-function Formulas*:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

*Note: Although they aren't listed for the other trigonometric functions, we can find them using the definitions of the six trigonometric functions above because they are all related to sine and cosine.

Trigonometric Unit Circle

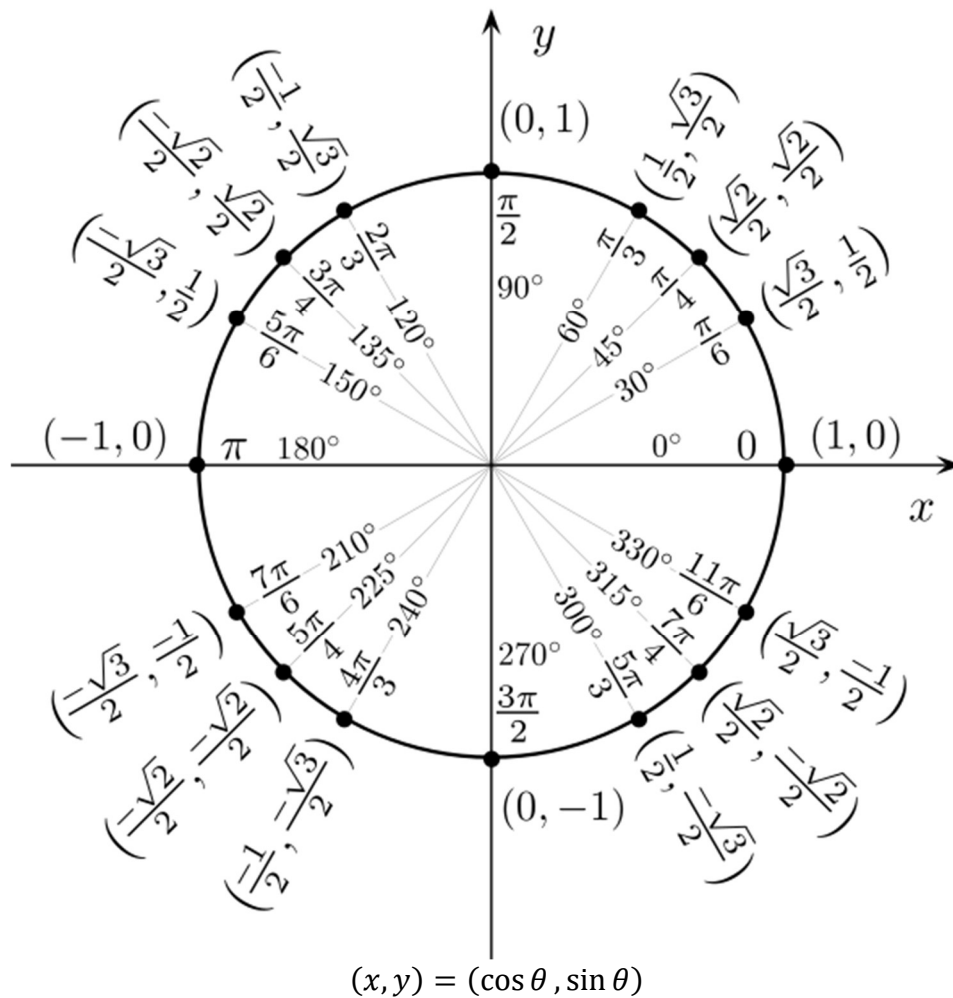


Table of Trigonometric Values of Common Angles of the Unit Circle:

Angle in Degrees / Radians	$\sin \theta$	$\cos \theta$	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
$30^\circ / \frac{\pi}{6}$	$1/2$	$\sqrt{3}/2$	$\frac{1}{1/2} = 2$	$\frac{1}{\sqrt{3}/2} = \frac{2\sqrt{3}}{3}$	$\frac{1/2}{\sqrt{3}/2} = \frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$
$45^\circ / \frac{\pi}{4}$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\frac{1}{\sqrt{2}/2} = \sqrt{2}$	$\frac{1}{\sqrt{2}/2} = \sqrt{2}$	$\frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$	$\frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$
$60^\circ / \frac{\pi}{3}$	$\sqrt{3}/2$	$1/2$	$\frac{1}{\sqrt{3}/2} = \frac{2\sqrt{3}}{3}$	$\frac{1}{1/2} = 2$	$\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$	$\frac{1/2}{\sqrt{3}/2} = \frac{\sqrt{3}}{3}$

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