Name: ____________________________________

School: ____________________________________

Grade: ____________________________________

2020 State Math Contest (Junior Exam)

Weber State University

March 4, 2020

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are not allowed on this exam.
- This is a multiple-choice test with 40 questions. Each question is followed by answers marked (a), (b), (c), (d), and (e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.
- You may not leave the room until at least 10:30 a.m.

---

1. You made a scale model of Earth and the moon. The Earth's diameter is about 1.3×10^4 kilometers. In the model the Earth's diameter is 50 centimeters. The moon's diameter is about 3.5×10^3 kilometers. Find the diameter of the moon in your model.

   (a) 12 centimeters
   (b) 13 centimeters
   (c) 14 centimeters
   (d) 15 centimeters
   (e) 16 centimeters

   Solution: (b) 13 centimeters. Set up a proportion: $\frac{13{,}000\,\text{km}}{50\,\text{cm}} = \frac{3500\,\text{km}}{x\,\text{cm}}$ $\rightarrow 13,000x = 50 \cdot 3500 \rightarrow 13,000x = 175,000 \rightarrow x = \frac{175{,}000}{13{,}000} \approx$ 13 centimeters.

2. What is the sum of all solutions (real, complex, and repeated) for the following polynomial equation: $x^5 - 4x^3 + 5x^2 + x - 7 = 0$.

   (a) 1
   (b) 0
   (c) -4
   (d) -7
   (e) None of the above

   Solution: (b) 0. As it is known, an equation of degree 5 has five solutions, counting all real, complex, and repeated roots. Let the solutions of the given polynomial equations be $x_1, ..., x_5$. Then $x^5 - 4x^3 + 5x^2 + x - 7 = (x - x_1)(x - x_2)(x - x_3)(x - x_4)(x - x_5)$.

   Next, by expanding the product and combining powers of x, we obtain:

   $x^5 - 4x^3 + 5x^2 + x - 7 = x^5 - (x_1 + x_2 + x_3 + x_4 + x_5)x^4 + ...+$ lower degree terms.

   By comparing the coefficients of x^4 from the both sides of the identity, we conclude that $x_1 + x_2 + x_3 + x_4 + x_5 = 0$.

3. How many squares are there in the infinite set {4, 44, 444, 4444, 44444, ...}?

   (a) One square
   (b) Two squares
   (c) Five squares
   (d) Eleven squares
   (e) Infinitely many squares

   Solution: (a) One square. 4 is a square. Every other number can be written as $4 \cdot X$ where $X \in \{11, 111, 1111, 11111...\}$. Note that each element of that set is three more than a multiple of 4 (e.g., $1111 = 1108 + 3$). However, odd squares are always one more than a multiple of 4, so none of the elements in that set are squares. This means every element in the set {44, 444, 4444, 44444, ...} is the product of a square (4) and another integer that is not a square. Such products cannot be squares.

4. An air traffic control radar screen is a circle with a diameter of 24 inches. The radar screen is set to have a scale of 6 inches : 25 nautical miles. What is the area of the circular region covered by the radar?

   (a) 157 nautical miles^2
   (b) 314 nautical miles^2
   (c) 7,850 nautical miles^2
   (d) 31,400 nautical miles^2
   (e) None of the above.

   Solution: (c) 7,850 nautical miles^2. The diameter of the radar screen is 24 inches, which is 6 inches $\times$ 4. The scale is 6 inches to 25 nautical miles, thus the screen covers a circle in diameter of 25 nautical miles $\times 4 = 100$ nautical miles. To find the area of a circle with a diameter of 100 nautical miles, we take the radius of 50 miles, square it, and multiply it by pi $\rightarrow A = \pi(50)^2 = (3.14)(2,500) = 7,850$ nautical miles^2.

5. Using a standard deck of 52 cards, how many ways are there to form a 5-card hand that has 3 cards of one rank and 2 cards of another rank.

   (a) 156
   (b) 468
   (c) 1,872
   (d) 3,744
   (e) 4,212

   Solution: (d) 3744. Multiply the number of ways to choose a rank that will have 2 cards by the number of ways to choose a rank that will have 3 cards by $C(4,2)$ and by $C(4,3)$. Thus, we have $13 \times 12 \times 6 \times 4 = 3,744$ ways.

6. Brady is tinkering with new chocolate milk recipes. He has made a 10% chocolate milk solution (this means the solution is 10% chocolate and 90% milk). He also has 30 gallons of a 25% chocolate milk solution. The 10% solution isn't very tasty, and the 25% solution is too chocolaty. Brady is convinced that a 15% solution will be perfect. Brady plans to add 10% solution to the 25% until it is a 15% solution. Assuming he has plenty of 10% solution, how many gallons of 15% solution can he make?

   (a) 10 gallons
   (b) 20 gallons
   (c) 40 gallons
   (d) 60 gallons
   (e) 90 gallons

   Solution: (e) 90 gallons. By adding the 10% solution to the 25% solution at a 2-to-1 ratio, we can get a 15% solution. In the 30 gallons of 25% solution there are 30 $\times$ 0.25 = 7.5 gallons of chocolate. Adding 60 gallons of 10% solution would add 30 $\times$ 0.1 = 3 more gallons of chocolate. The total would be 13.5 gallons of chocolate in a total of 90 gallons of chocolate milk: 13.5 $\div$ 90 = 0.15.

7. A ball is released from a height of 10 feet. Each time the ball hits the ground it will bounce directly back up $\frac{3}{4}$ of the distance it has fallen. What is the total distance the ball travels before it stops bouncing?

   (a) 70 feet
   (b) 80 feet
   (c) 60 feet
   (d) 75 feet
   (e) None of the above

   Solution: (a) 70 feet. The total distance is $10 + 2(\frac{3}{4})10 + 2(\frac{3}{4})^2 10 + 2(\frac{3}{4})^3 10 + \cdots = 10 + 2(\frac{3}{4})(10 + (\frac{3}{4})10 + (\frac{3}{4})^2 10 + \cdots) = 10 + \frac{3}{2}\sum_{n=1}^{\infty} 10(\frac{3}{4})^{n-1} = 10 + \frac{3}{2}\left(\frac{10}{1-\frac{3}{4}}\right) = 70$ feet.

8. The function $f(x) = -\frac{2x}{x-1}$ is one-to-one. Its inverse is

   (a) $f^{-1}(x) = \frac{x}{x+2}$
   (b) $f^{-1}(x) = \frac{x-1}{-2x}$
   (c) $f^{-1}(x) = \frac{-2x}{y} + 1$
   (d) $f^{-1}(x) = \frac{x}{2(x+1)}$
   (e) $f^{-1}(x) = \frac{-2y+x}{x}$

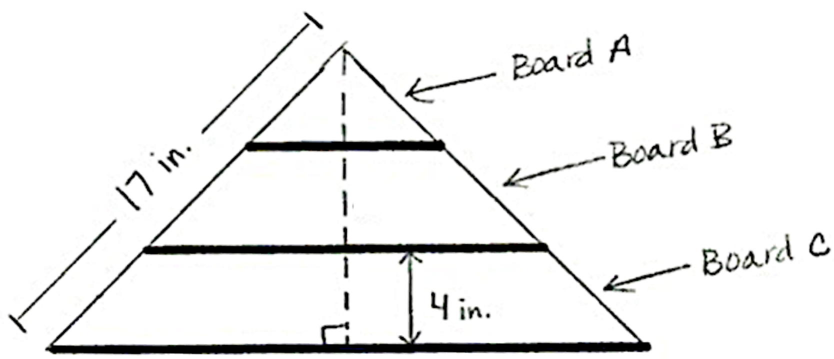
   Solution: (a) $f^{-1}(x) = \frac{x}{x+2}$.
   1) Switch x and y: $x = -\frac{2y}{y-1}$.
   2) Solve for y: $xy - x = -2y \to xy + 2y = x \to y(x+2) = x \to y = \frac{x}{x+2}$.
   3) Replace y with $f^{-1}(x) = \frac{x}{x+2}$.

9. A palindrome is a number that remains the same even if you reverse the order of its digits. For example, the number 15,351 is a palindrome. What is the largest positive factor of every four-digit palindrome?

   (a) 1
   (b) 2
   (c) 7
   (d) 9
   (e) 11

   Solution: (e) 11. Let $1000a + 100b + 10b + a$ be any 4-digit palindrome.
   $1000a + 100b + 10b + a = 1001a + 110b = 11(91a + 10b)$, hence 11 is a factor.

10. Jason wants to build a corner shelf for his living room. Rather than building the shelf out of a solid piece of wood, he plans to construct it out of three 4-inch wide boards, as shown in the diagram below.



   If he wants the total length of the shelf along the wall sides to be 17 inches, what will the length of the **front** edge of the shelf (indicated by the **bold** line on board C) be? Round to the nearest inch.

   (a) 17 inches
   (b) 24 inches
   (c) 578 inches
   (d) 12 inches
   (e) 20 inches

   Solution: (b) 24 inches. Since the shelf is going into a corner of a house, you may assume the shelf is in the shape of a right triangle. Thus, $17^2 + 17^2 = 578$. Hence, the front edge of board C is $\sqrt{578} \approx 24$ inches.

11. Using the diagram in the previous problem (#10), what is the length of the **front** edge of boards B and A (indicated by the **bold** lines on each board), in that order? Round to the nearest inch.

    (a) 8 inches, 4 inches
    (b) 9 inches, 5 inches
    (c) 13 inches, 7 inches
    (d) 16 inches, 8 inches
    (e) Not enough information to compute.

    Solution: (d) 16 inches, 8 inches. The depth of the shelf (indicated by the height of the triangle marked in the picture) is 12 inches. Since the boards are known to be 4 inches wide, then by similar triangles, $\frac{12}{24} = \frac{8}{x}$. Hence, $x = 16$ inches is the length of the front edge of board B. Similarly, for board A, we have $\frac{12}{24} = \frac{4}{x}$. So $x = 8$ inches.

12. A committee is to be formed consisting of 4 women and 3 men. There are 12 women and 8 men available for the committee to be formed. How many possible committees are possible?

    (a) 495
    (b) 56
    (c) 390,700,800
    (d) 27,720
    (e) 77,520

    Solution: (d) 27,270. The general multiplication rule tells us that this is the number of ways to select 4 women out of 12 times the number of ways to select 3 men out of 8 in each case without regard to order. Hence, using combinations there are $C(12,4) \times C(8,3) = 27,720$.

13. A trader purchased a bracelet at a certain price. The trader then marked up the price 70% and attempted to sell it at that price. The bracelet sat in the case for days before the trader cut the sale price by 25%. The next day someone bought the bracelet. What percent profit did the trader make?

    (a) 2.8%
    (b) 17.5%
    (c) 27.5%
    (d) 35.7%
    (e) 45%

    Solution: (c) 27.5%. Let the price the trader paid for the bracelet be $\$X$. Then the original sale price is given by $X \cdot 1.7$. The reduced sale price is given by $X \cdot 1.7 \cdot 0.75 = X \cdot 1.275$. Profit is given by $\frac{1.275X - X}{X} = \frac{X(1.275-1)}{X} = 0.275 = 27.5\%$.

14. List all the possible values of $\frac{x}{|x|} + \frac{y}{|y|} + \frac{xy}{|xy|}$, where $x, y \in \mathbb{R}$.

(a) $\{-3, 3\}$
(b) $\{-3, 0, 3\}$
(c) $\{-3, -1, 1, 3\}$
(d) All real numbers
(e) None of above

Solution: (c) $\{-3, -1, 1, 3\}$. $\frac{a}{|a|} = \pm 1$

15. Find the units digit of 3^{999}.

(a) 1
(b) 3
(c) 5
(d) 7
(e) 9

Solution: (d) 7. We see that the units digits for powers of 3 follow a pattern:

Exponent	Units Digit
1	3
2	9
3	7
4	1
5	3
6	9
7	7
8	1

The pattern repeats in groups of 4. $999 = 249 \cdot 4 + 3$ so 999 is three more than a multiple of 4, hence the units digit of 3^{999} is 7.

16. Washington School sent their 4 best chess players to challenge the 4 best chess players from Lincoln School. Each of the 4 Washington players will be randomly paired with a different Lincoln player for the first round of games. How many different possible pairings are possible for the first round of games?

    (a) 4
    (b) 16
    (c) 24
    (d) 576
    (e) 40,320

    Solution: (c) 24. Imagine that there are 4 tables with chessboards set up, and that each of the Lincoln players is already seated at a table when the Washington players arrive. The order of the Lincoln players is unimportant. The first Washington player is randomly assigned to one of the 4 seated Lincoln players. The next Washington player is randomly assigned to one of the 3 remaining Lincoln players. The next Washington player is assigned to one of the 2 remaining Lincoln players. And the last Washington player has only one choice of Lincoln opponent. Therefore, the total number of pairings is $4 \times 3 \times 2 \times 1 = 24$.

17. For all real numbers b and c such that the product of c and 3 is b, which of the following expressions represent the sum of c and 3 in terms of b?

    (a) $b + 3$
    (b) $3(b + 3)$
    (c) $3b + 3$
    (d) $\frac{b}{3} + 3$
    (e) $\frac{b+3}{3}$

    Solution: (d) $\frac{b}{3} + 3$. We know that $3c = b$ so $c = \frac{b}{3}$ which makes $c + 3 = \frac{b}{3} + 3$.

18. If $a < 0$, then the quadratic function $y = x^2 + ax + a$ has

    (a) One zero
    (b) Two real zeros
    (c) None
    (d) Two conjugate complex zeros
    (e) None of above

    Solution: (b) Two real zeros. Let $\Delta = a(a - 4)$ must be positive for $a < 0$.

19. What is the largest possible value for the sum of two fractions such that each of the four 1-digit prime numbers occurs as one of the numerators or denominators?

    (a) $\frac{3}{2} + \frac{7}{5}$
    (b) $\frac{5}{7} + \frac{2}{3}$
    (c) $\frac{5}{2} + \frac{7}{3}$
    (d) $\frac{7}{2} + \frac{5}{3}$
    (e) $\frac{9}{5} + \frac{7}{3}$

    Solution: (d) $\frac{7}{2} + \frac{5}{3}$. For the largest possible sum, we need the smallest possible denominators. Thus, $\frac{7}{2} + \frac{5}{3}$ and $\frac{5}{2} + \frac{7}{3}$ are the only possible options. Of the two, the set with the greatest differential will have the greatest sum, which is $\frac{7}{2} + \frac{5}{3}$.

20. The red part of Santa's hat is made from material that forms a cone. If the cone has a base diameter of 10 inches and a height of 12 inches, how much material is needed to make the red part of Santa's hat?

    (a) 78.5 inches^2
    (b) 188 inches^2
    (c) 204 inches^2
    (d) 314 inches^2
    (e) 408 inches^2

    Solution: We need to find the surface area of a cone without the base. The formula for the surface area of the cone without the base is $\pi r l$, where r is the radius of the cone and l is the slant height of the cone. The radius is 5 inches, and the slant height is $\sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2} = 13$ inches $\rightarrow (3.14)(5)(13) \approx 204$ inches^2.

21. The point $P = (-5, 5)$ is reflected over the line $y = 3x$, resulting in a point Q. What is the x-coordinate of point Q?

    (a) 4
    (b) 5
    (c) 6
    (d) 7
    (e) 8

    Solution: (d) 7. In fact, we shall see that $Q = (7, 1)$. The line through $(-5, 5)$ that is perpendicular to $y = 3x$ is $y - 5 = (-1/3)(x+5)$, or equivalently $y = (-1/3)(x+5)+5$. The intersection of this line with $y = 3x$ is at the point $(1, 3)$. To move from the point $(-5, 5)$ to $(1, 3)$, we must move down 2 and to the right 6. From the point $(1, 3)$, if we move down 2 additional units and right 6 additional units, we arrive at the point $(7, 1)$.

22. Two teachers and three students sit randomly around a round table. What is the probability each student sits next to at least one teacher?

   (a) 50%
   (b) 60%
   (c) 40%
   (d) 100%

   Solution: (a) 50%. Number the seats 1, 2, 3, 4, and 5, and treat the two teachers and the three students as indistinguishable from one another, respectively. There are $\frac{5!}{3!2!} = 10$ ways for them to seat. The only way a student is not next to at least one teacher is that they sit on the seats 1, 2, and 3, seats 2, 3, and 4, seats 3, 4, 5, seats 4, 5, 1, or seats 5, 1, and 2. So there is 5/10=50% chance a student is not sitting next to a teacher, hence, there is a 50% chance each student sits next to at least one teacher.

23. A ladder is leaning against a vertical wall with its bottom 15 feet from the wall. If the bottom of the ladder is pulled out 9 feet farther away from the wall, its upper end slides 13 feet down. What is the length of the ladder?

   (a) 25 feet
   (b) 8 yards
   (c) 30 feet
   (d) 20 feet
   (e) 7 yards

   Solution: (a) 25 feet. Let l be the length of the ladder and h the height to which it extends initially. Then $h^2 + 15^2 = l^2$ and $(h - 13)^2 + (15 + 9)^2 = l^2$. Subtracting the 2nd equation from the 1st, we get $26h - 13^2 - 24^2 + 15^2 = 0$. That is $h = 20$. Since $l > 0$, the solution of $l^2 = 20^2 + 15^2$ is $l = 25$.

24. If $(1, 2) \in A \cap B, A = \{(x, y)|ax + by = 1\}, B = \{(x, y)|2ax + by^2 = 2\}$, then $a + b =$

   (a) 3
   (b) 1
   (c) -1
   (d) All real numbers
   (e) None of above

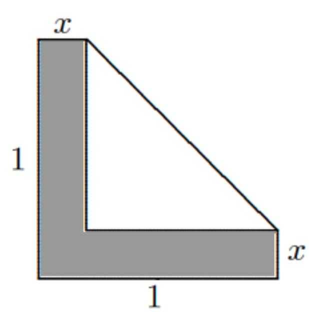
   Solution: (d) All real numbers. Only restriction is, $a + 2b = 1$

25. Note that $5625 = 3^2 \cdot 5^4$. How many positive integer divisors does 5625 have?

(a) 6
(b) 7
(c) 15
(d) 16
(e) None of the above.

Solution: (c) 15. The integer divisor must have 3 to a power and 5 to a power. The possibilities for the power on 3 are 0, 1, and 2, which are three choices. The possibilities for the power on 5 are 0, 1, 2, 3, and 4, which are five choices. The total number is $3 \times 5 = 15$ choices.

26. In the figure below, if the area of the letter L part equals the area of the triangle, and the length of the base and the height is 1 unit, what is the length x of the ends of the letter L?



(a) $\sqrt{\frac{1}{3}}$ units
(b) $\sqrt{\frac{2}{3}}$ units
(c) $1-\sqrt{\frac{1}{3}}$ units
(d) $1-\sqrt{\frac{2}{3}}$ units
(e) $1+\sqrt{\frac{2}{3}}$ units

Solution: (d) $1-\sqrt{\frac{2}{3}}$ units. Adjoining the reflection of the triangle across its hypotenuse to the figure yields a square. Now, the area of the triangle must equal $\frac{1}{3}$ the area of the square. Thus, $\frac{1}{2}bh = \frac{1}{3}s^2 \to \frac{1}{2}(1-x)(1-x) = \frac{1}{3}(1)^2 \to \frac{1}{2}(1-2x+x^2) = \frac{1}{3}$ $\to 1-2x+x^2 = \frac{2}{3} \to \frac{1}{3} - 2x + x^2 = 0 \to x = (1-\sqrt{\frac{2}{3}}), (1+\sqrt{\frac{2}{3}})$. Since x must be less than one, $x = 1-\sqrt{\frac{2}{3}}$.

27. It is known that in a certain population 10% of the population is afflicted by disease A, 15% are afflicted by disease B, and 5% are afflicted by both diseases. What proportion of the population is afflicted by disease B if they are known to have disease A?

   (a) 50%
   (b) 20%
   (c) 5%
   (d) 33.33%
   (e) 1.5%

   Solution: (a) 50%. The problem asked for the conditional probability of B, given A, which is $\frac{P(A \,\mathrm{and} B)}{P(A)} = \frac{0.05}{0.1} = 50\%$.

28. Find the exact value of $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$.

   (a) 2
   (b) 1
   (c) -2
   (d) 3
   (e) None of the above

   Solution: (a) 2. Let $x = \sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$. Then $x > 0$ and $x^2 = 3 + 2\sqrt{2} - 2\sqrt{9-8} + 3 - 2\sqrt{2} = 4$. Thus, $x = 2$.

29. A temperature of 0° Celsius is equivalent to 32° Fahrenheit. At what temperature, if any, will a Fahrenheit thermometer and Celsius thermometer have the same reading? Assume that water boils at a temperature of 100°C (212°F).

   (a) $-40°$
   (b) $-10°$
   (c) 20°
   (d) 40°
   (e) There is no temperature when the two scales produce the same reading.

   Solution: (a) $-40°$. The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. We can obtain this formula by finding the equation of a line that goes through the points $(0, 32)$ and $(100, 212)$. To answer the question, we set $C = \frac{9}{5}C + 32$. Solving this equation yields $C = -40°$.

30. A rhombus has sides of length 10 units, and its diagonals differ by 4 units. What is its area?

    (a) 12 units^2
    (b) 24 units^2
    (c) 48 units^2
    (d) 96 units^2
    (e) 192 units^2

    Solution: (b) 96 units squared. The diagonals of a rhombus are perpendicular. If b denotes half the length of the shorter diagonal, then the rhombus is composed of four right triangles with bases b and $b+2$ and hypotenuse 10. Thus, $b^2+(b+2)^2=100$, hence $b^2+2b-48=0$, so $b=6$ units. The area of each of the triangles is $\frac{6\cdot 8}{2}=24$ units squared and the total is 96 units squared.

31. In a group of five friends, the sum of the ages of each group of four of them are 89, 90, 92, 94, and 95. What is the age of the youngest?

    (a) 18
    (b) 20
    (c) 21
    (d) 24
    (e) None of the above

    Solution: (b) 20. The sum of these given five numbers will include age of each one four times. Thus, the sum of their ages is $(89+90+92+94+95)/4=115$. So the age of the youngest one is $115-95=20$.

32. John wants to buy a backpack but he is \$18 short. For the same backpack, Kate is \$7 short, Nancy is \$6 short, and Bubba is \$4 short. Which two of them, combining their money, can **certainly** buy the backpack?

    (a) John and Kate
    (b) John and Nancy
    (c) Nancy and Bubba
    (d) All the above.
    (e) None of the above

    Solution: (c) Nancy and Bubba. Let x be the price for the backpack. Since John is \$18 short, $x \geq 18$, or $x-10 \geq 8$. Next, Kate has $x-7$ dollars, Nancy has $x-6$ dollars and Buba has $x-4$ dollars. Nancy and Bubba combined will have $2x-10$ dollars, that is $2x-10=x-10+x \geq 8+x > x$. Hence, they will have more than the price of the backpack x. However, if the backpack costs \$19, then neither John and Kate, who would have \$13 combined, nor John and Nancy, who would have \$14 combined, would not have enough funds to buy the backpack.

33. In the sequence of numbers $1, 4, 3, -1, ...$ each term after the first two is equal to the term preceding it minus the term preceding that. Find the sum of the first one hundred terms of the sequence.

(a) 1
(b) -2
(c) -3
(d) 7
(e) 3

Solution: (d) 7. The sequence is periodic of length 6, $(1, 4, 3, -1, -4, -3, 1, ...)$. The sum of any six consecutive terms is zero. Since 96 is a multiple of six, the sum of the first 96 terms is zero and the sum of the first hundred terms is the same as the sum of the first 4 terms. $1 + 4 + 3 + (-1) = 7$.

34. Square $ABCD$ has sides of length 3 units. Side AB is extended through B to E with $BE = 1$ unit. Segment DE intersects side BC at point F. What is the area of the triangle CDF?

(a) $\frac{27}{8}$ units^2
(b) 3 units^2
(c) 4 units^2
(d) $\frac{25}{6}$ units^2
(e) 6 units^2

Solution: (a) $\frac{27}{8}$ units^2. By similar triangles, $\frac{BF}{BE} = \frac{AD}{AE} = \frac{3}{4}$. So, $BF = \frac{3}{4}$ and $CF = 3 - \frac{3}{4} = \frac{9}{4}$. Thus, the area of the triangle CDF is $\frac{1}{2} \times 3 \times \frac{9}{4} = \frac{27}{8}$ units^2.

35. Each box of cereal contains a toy. The toy comes in 4 different colors: red, orange, green, or blue. Each of the 4 colors is equally likely to occur. Alex wants to get a toy in each of the 4 colors. If Alex buys 4 boxes of cereal, what is the probability that they get one toy of each of the 4 colors?

(a) $\frac{1}{256}$
(b) $\frac{1}{35}$
(c) $\frac{3}{32}$
(d) $\frac{1}{4}$
(e) $\frac{3}{4}$

Solution: (c) $\frac{3}{32}$. No matter what color is in the first box opened by Alex, it contains a color he doesn't have. The probability that the second box has a new color is $\frac{3}{4}$. If Alex now has 2 different colors, the probability that the third box contains a color he doesn't have is $\frac{2}{4}$. Then, the probability that the 4th box contains the missing color is $\frac{1}{4}$. Since the probabilities are independent, to find the overall probability, multiply the 4 probabilities together: $1 \times \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{6}{64} = \frac{3}{32}$.

36. A goat, a horse, and a cow mistakenly enter a farmer's wheat field and eat some stalks of wheat. The horse eats twice as many stalks as the goat, and the cow eats twice as many stalks as the horse. The farmer demands 5 tou of wheat from the owners of the animals to replace what was eaten. How much wheat should be replaced by the horse's owner?

   (a) $\frac{5}{7}$ tou
   (b) 1 tou
   (c) $\frac{10}{7}$ tou
   (d) 2 tou
   (e) $\frac{20}{7}$ tou

   Solution: (c) $\frac{10}{7}$ tou. Let x be the the amount of wheat (in tou) that should be replaced by the goat's owner. Then, based on the problem description, we can form the equation $x + 2x + 4x = 5$. Thus, $x = \frac{5}{7}$. Since the horse ate twice as much as the goat, the owner should replace $\frac{10}{7}$ tou worth of wheat.

37. There are three, three-digit positive integer numbers. None of them contains the digit 0, but all the remaining nine digits 1, 2 ,..., 9, appear exactly once in each number. Suppose that, from each given number, a new number is formed where the first and the last digits are exchanged. For example, if the three given numbers were 267, 813, 594, then the three new numbers would be 762, 318, 495. Now, assume that the sum of the three given numbers is 1665. What will be the sum of the three new numbers with the first and third digits exchanged?

   (a) 5661
   (b) 4995
   (c) 1665
   (d) 999
   (e) None of above

   Solution: (c) 1665. Since the last digit of the sum is 5, the sum of the last digits of those three numbers must be 5, 15 or 25. One can verify that it is impossible to form 5 or 25 by adding three distinct digits between 1 and 9. Hence the sum of the last three digits is 15. But then, the sum of second digits of those three numbers also must be 15. (The second digit of the sum of all three numbers is 6.) And so is the sum of the first three digits of the three numbers. By interchanging the first and the third digit will not change the sum of digits. Hence, the total sum will be the same 1665.

38. A positive whole number leaves a remainder of 7 when divided by 11 and a remainder of 10 when divided by 12. What is the remainder when divided by 66?

    (a) 0
    (b) 40
    (c) 28
    (d) 52
    (e) 29

    Solution: (b) 40. $n = 11a + 7 = 12b + 10$. So the first several possibilities for $11a + 7$ are 7, 18, 29, 40, 51, 62, 73, 84, 95, 106, 117, ..., and for $12b + 10$ we get 10, 22, 34, 46, 58, 70, 82, 94,106,118,... These match at 106 and $106 = 66 \cdot 1 + 40$. Thus, the remainder is 40.

39. Consider the square $ABCD$ and extend side AB through B to point E. Segment DE intersects side BC at point F. If the ratio of area of the triangle EBF to the area of the triangle CDF is $\frac{1}{9}$, what is the ratio of the length of segment BE to the length of side AB?

    (a) $\frac{1}{3}$
    (b) $\frac{1}{2}$
    (c) $\frac{3}{2}$
    (d) $\frac{2}{3}$

    Solution: (a) $\frac{1}{3}$. By similar triangles, $\frac{BF}{DF} = \frac{BE}{DC}$. Then, $\frac{1}{9} = \frac{\text{area of the triangle } EBF}{\text{area of the triangle } CDF} = \frac{\frac{1}{2} \cdot BF \cdot BE}{\frac{1}{2} \cdot DF \cdot DC} = \frac{BF \cdot BE}{DF \cdot DC} = \frac{(BE)^2}{(DC)^2} = \frac{(BE)^2}{(AB)^2}$. So, $\frac{BE}{AB} = \frac{1}{3}$.

40. A 1-meter measuring stick is cut at a randomly selected location, creating two sticks. What is the probability that the larger of the two sticks is over twice as long as the shorter of the two sticks?

    (a) $\frac{1}{4}$
    (b) $\frac{1}{3}$
    (c) $\frac{1}{2}$
    (d) $\frac{2}{3}$
    (e) $\frac{3}{4}$

    Solution: (d) $\frac{2}{3}$. Imagine the meter stick labeled with markings from 0 to 1. If the cut is made at any location between 0 and $\frac{1}{3}$, the smaller stick will be less than $\frac{1}{3}$ meter and the larger stick would be larger than $\frac{2}{3}$, satisfying the criteria. The same will be true if the cut is made at any location between $\frac{2}{3}$ and 1. Thus, in $\frac{2}{3}$ of the possible cut locations, the criteria is met.