State Math Contest 2019 Junior Exam
Weber State University
March 5, 2019

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are not allowed on this exam.
- This is a multiple choice test with 40 questions. Each question is followed by answers marked a), b), c), d), and e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.
- You may not leave the room until at least 10:15 a.m.

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1. Find the product of all real solutions to the equation $x^4 - 10x^2 - 24 = 0$.

(a) –24
(b) –12
(c) –2
(d) 2
(e) 12

Correct Answer: (b) –12.
Solution: $x^4 - 10x^2 - 24 = (x^2 + 2)(x^2 - 12) = 0$. If $x^2 - 12 = 0$, $x = \pm\sqrt{12}$. If $x^2 + 2 = 0$, there are no real solutions. The product of the real solutions is –12.

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2. Joe can mop the kitchen floor in 12 minutes. Liz can mop the kitchen floor in 6 minutes. If Joe, Liz, and Zack work together, they can mop the kitchen floor in 2 minutes and 40 seconds. How long does it take Zack to mop the kitchen floor by himself?

(a) 4 minutes
(b) 6 minutes
(c) 8 minutes
(d) 10 minutes
(e) 12 minutes

Correct Answer: (c) 8 minutes
Solution: Joe takes 12 minutes, Liz takes 6 minutes, Zack takes x minutes, and together they take 2 minutes and 40 seconds (i.e., $2\frac{2}{3}$ minutes or $\frac{8}{3}$ minutes). We form the following equation and solve for x:

$\frac{1}{12} + \frac{1}{6} + \frac{1}{x} = \frac{1}{\frac{8}{3}}$
$\frac{1}{12} + \frac{1}{6} + \frac{1}{x} = \frac{3}{8}$
$8x\left(\frac{1}{12} + \frac{1}{6} + \frac{1}{x}\right) = \left(\frac{3}{8}\right)8x$
$\frac{8}{12}x + \frac{8}{6}x + 8 = 3x$
$\frac{4}{6}x + \frac{8}{6}x + 8 = 3x$
$\frac{12}{6}x + 8 = 3x$
$2x + 8 = 3x$
$8 = x$

Thus, Zack takes 8 minutes to mop the kitchen floor.

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3. The complex number $3 - 2i$ is a root of the polynomial $5x^3 - 32x^2 + 77x - 26$. What is the imaginary part of the other complex root of this polynomial?

(a) $\frac{2}{3}$
(b) 1
(c) $\frac{3}{2}$
(d) 2
(e) 3

Correct Answer: (d) 2.
Solution: Whenever a complex number is the root of a *real* polynomial, then the complex conjugate of that number must also be a root of that polynomial. Therefore, since $3 - 2i$ is a root, it follows that $3 + 2i$ (which has imaginary part 2) must also be a root.

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4. Suppose $x + \frac{1}{y} = 2$ and $y + \frac{1}{x} = 4$. What is the value of $\frac{y}{x}$?

(a) $\frac{1}{2}$
(b) 2
(c) $\frac{1}{4}$
(d) 4
(e) 1

Correct Answer: (b) 2
Solution: Multiply the first equation by y and the second by x to get $xy + 1 = 2y$ and $xy + 1 = 4x$. Hence $2y = 4x$ and so $\frac{y}{x} = 2$.

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5. The ratio of vinegar to water in three separate solutions of vinegar and water is 1:1, 1:2, and 1:3. If equal quantities of each solution are combined together and mixed, what will be the ratio of vinegar to water in the resulting solution?

(a) 1:6
(b) 3:6
(c) 7:12
(d) 11:18
(e) 13:23

Correct Answer: (e) 13:23
Solution: Assume that you have 1 cup of each solution. The first would have $\frac{1}{2}$ cup of vinegar and $\frac{1}{2}$ cup of water. The second would have $\frac{1}{3}$ cup of vinegar and $\frac{2}{3}$ cup of water. The third would have $\frac{1}{4}$ cup of vinegar and $\frac{3}{4}$ cup of water. The total amount of vinegar is $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$. The total amount of water is $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{23}{12}$. Thus, the ratio of vinegar to water is 13:23.

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6. On a "prime day," both the month and the day are prime. For example, March 13th is a prime day, because March is month 3, which is a prime, and so is 13. How many prime days are there in 2019?

(a) 52
(b) 53
(c) 55
(d) 50
(e) None of the above.

Correct Answer: (a) 52
Solutions: Prime months are February (2), March (3), May (5), July (7), and November (11). Prime days are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31. February has 9 prime days. March, May, and July have 11 prime days each. November has 10 prime days. Thus, the total is 52 prime days. (Consequently, in a leap year, there are 53 prime days because February 29th is a prime day, and 53 is also prime!)

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7. A new car is being developed that uses its entire fuel supply in 38 hours when left idling. However, when driven at 60 mph on a test track, the car uses about three-and-a-half times as much fuel per hour as it does when idling. Suppose the car has been idling for 10 hours, and then it is run at 60 mph on the test track. How much longer will the car run before it uses up all of its fuel?

(a) 28 hours
(b) 10 hours
(c) 8 hours
(d) 7 hours
(e) 3 hours

Correct Answer: (c) 8 hours
Solution: Let I be the number of hours the car idles, and R be the number of hours the car runs. The situation is modeled by the equation $I = 3.5R$. After idling for 10 hours, the car has 28 hours of idling left. Substituting in the equation yields $28 = 3.5R$, which means $R = 8$.

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8. In order to buy a $125,000 house, a couple puts down $25,000 and takes out a mortgage loan on the balance from a bank. Their mortgage payment is $877.57 per month for 360 months. To pay off the mortgage loan, they pay $877.57 per month for the following 360 months. When the mortgage loan is fully paid off, how much more will they end up paying for the house than the original price? (Round you answer to the nearest whole number.)

(a) $215,925
(b) $216,080
(c) $315,925
(d) $340,925
(e) $341,080

Correct Answer: (a) $215,925
Solution: $125,000 – $25,000 = $100,000 balance for mortgage
($877.57)(360 months) = $315,925.20 total mortgage payments
$315,925.20 – $100,000 = $215,925.20, which is the amount of money paid in excess of the original price.

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9. What is the remainder when 2^{1000} is divided by 13?

(a) 2

(b) 3
(c) 4
(d) 10
(e) 12

Correct Answer: (b) 3
Solution:
When we divide 2^0 by 13, remainder is 1
When we divide 2^1 by 13, remainder is 2
When we divide 2^2 by 13, remainder is 4
When we divide 2^3 by 13, remainder is 8
When we divide 2^4 by 13, remainder is 3
When we divide 2^5 by 13, remainder is 6
When we divide 2^6 by 13, remainder is 12
When we divide 2^7 by 13, remainder is 11
When we divide 2^8 by 13, remainder is 9
When we divide 2^9 by 13, remainder is 5
When we divide 2^{10} by 13, remainder is 10
When we divide 2^{11} by 13, remainder is 7
When we divide 2^{12} by 13, remainder is 1

Now the cycle of remainders has started to repeat again i.e. 1, 2, 4, 8, 3, 6, 12 … until we have 1 again. We also know that every power that is a multiple of 12 will have a remainder of 1. We also know that if the power is $12n + m$ (where n is a whole number and m is a whole number less than 12) the remainder will correlate with the remainder for 2^m. Since $1000 = 12 \times 83 + 4$, and is of the form $12n + 4$, we know that the remainder will be the same as it was for 2^4. Thus the remainder is 3.

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10. You are driving to a job interview and do not want to be late. You figure you must average 60 miles per hour for the remaining two miles to the interview to be on time. Due to an accident, you average 30 miles per hour for the first mile. What speed do you need to average for the second mile to make it to your interview on time?

(a) 90 miles per hour
(b) 30 miles per hour
(c) 120 miles per hour
(d) 105 miles per hour
(e) It is impossible to be on time to the interview

Correct Answer: (e) It is impossible to be on time to the interview
Solution: Averaging 60 mph for 2 miles takes exactly 2 minutes. If you average 30 mph for the first mile it takes exactly 2 minutes to drive the first mile. This leaves no time to drive the second mile and be on time.

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11. How many positive integers less than 50 have an odd number of different factors.

(a) 7
(b) 11
(c) 13
(d) 17
(e) None of the above

Correct Answer: (a) 7.
Solution: The numbers with odd number of different factors are: 1, 4, 9, 16, 25, 36, 49 so there are seven numbers.
For example, 4 has factors: 1, 2, 4
9 has factors: 1, 3, 9
16 has factors: 1, 2, 4, 8, 16

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12. If three fair dice are rolled, what is the probability that the sum of the three dice is 6?

(a) $\frac{5}{108}$
(b) $\frac{1}{18}$
(c) $\frac{1}{24}$
(d) $\frac{10}{36}$
(e) $\frac{1}{27}$

Correct Answer: (a) $\frac{5}{108}$
Solution: In rolling 3 dice, there are $6^3 = 216$ possible outcomes, 6 possibilities on each dice. Here are the possible ways to get a sum of six: (1,1,4); (1,4,1); (1,2,3); (1,3,2); (2,1,3); (2,3,1) (2,2,2); (3,2,1); (3,1,2); and (4,1,1). Total possible for sum of six is 10. Therefore probability is $\frac{10}{216} = \frac{5}{108}$.

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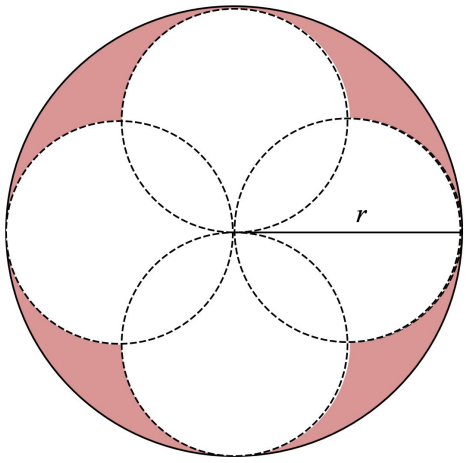
13. The largest circle pictured has a radius $r = 49$ units. What is the area of the shaded region? (For simplicity, use $\pi = \frac{22}{7}$.)



(a) 2401 units^2

(b) 1372 units^2
(c) 2744 units^2
(d) 7546 units^2
(e) 5488 units^2

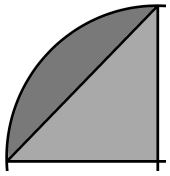
Correct Answer: (b) 1372 units^2
Solution: The area of the shaded region is equal to the area of the largest circle minus the area of the 4 overlapping circles.
The area of the largest circle is: πr^2

$$= \tfrac{22}{7} \cdot 49^2$$
$$= \tfrac{22}{7} \cdot 49 \cdot 49$$
$$= 22 \cdot 7 \cdot 49$$
$$= 7546 \text{ units}^2$$

The area of the 4 overlapping circles is: 4 ▪ area of the small circle – 4 ▪ area of the lenses
The area of the small circle is: πr^2

$$= \tfrac{22}{7} \cdot \left(\tfrac{49}{2}\right)^2$$
$$= \tfrac{22}{7} \cdot \tfrac{49}{2} \cdot \tfrac{49}{2}$$
$$= \tfrac{22}{1} \cdot \tfrac{7}{2} \cdot \tfrac{49}{2}$$
$$= \tfrac{7546}{4}$$

The area of 4 small circles is: $4 \cdot \tfrac{7546}{4} = 7546 \text{ units}^2$
The area of one lens is 2 ▪ area of $\tfrac{1}{2}$ of a lens.
The area of $\tfrac{1}{2}$ of a lens is the area of $\tfrac{1}{4}$ of the small circle – the area of a triangle:



The area of $\tfrac{1}{4}$ of the small circle is: $\tfrac{1}{4} \cdot \tfrac{7546}{4} = \tfrac{7546}{16}$ units^2
The area of the triangle is: $\tfrac{1}{2}bh$

$$= \tfrac{1}{2} \cdot \tfrac{49}{2} \cdot \tfrac{49}{2}$$
$$= \tfrac{2401}{8} \text{ units}^2$$

The area of $\tfrac{1}{2}$ of a lens is: $\tfrac{7546}{16} - \tfrac{2401}{8} = \tfrac{7546}{16} - \tfrac{4802}{16} = \tfrac{2744}{16}$ units^2
The area of a lens is: $\tfrac{2744}{16} \cdot 2 = \tfrac{2744}{8} = 343 \text{ units}^2$
The area of 4 lenses is: 343 ▪ 4 = 1372 units^2
The area of the 4 overlapping circles is: 7546 – 1372 = 6174 units^2
The shaded area is: 7546 – 6174 = 1372 units^2

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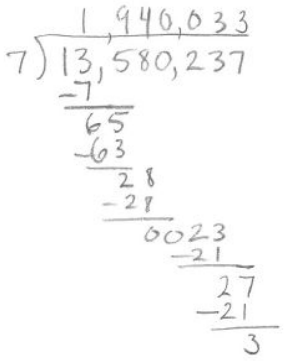
14. Which of the following numbers is a factor of 13,580,237?

(a) 3
(b) 6
(c) 9
(d) 7

(e) 11

Correct Answer: (e) 11
Solution: Using divisibility rules, the number is not divisible by 3 because the sum of the digits is 29, which is not divisible by 3. The number is not divisible by 6, because it is not divisible by 3. The number is not divisible by 9, because it is not divisible by 3. The number is not divisible by 7 because



Thus, the number must be divisible by 11.

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15. Write 0.224 in base 5.

(a) 0.110_5
(b) 0.103_5
(c) 0.224_5
(d) 0.112_5

Correct Answer: (b) 0.103_5.
Solution: $0.224 = 0.abc_5 \rightarrow 0.224 = \frac{a}{5} + \frac{b}{25} + \frac{c}{125} \rightarrow 1.12 = a + \frac{b}{5} + \frac{c}{25} \rightarrow a = 1$ and $0.12 = \frac{b}{5} + \frac{c}{25} \rightarrow 0.6 = b + \frac{c}{5} \rightarrow b = 0$ and $0.6 \rightarrow \frac{c}{5} \rightarrow 3 = c$.

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16. If the angles of a right triangle are in *arithmetic progression* then the ratio of the smallest side is to the largest side is:

(a) $\frac{1}{3}$
(b) $\frac{1}{4}$
(c) $\frac{1}{2}$
(d) $\frac{2}{3}$

Correct Answer: (c) $\frac{1}{2}$
Solution: Let ABC be the right triangle with $A = 90$ degrees. If the smallest angle, let's say B has x degrees and since they are in arithmetic progression, with common difference h degrees, then $x + (x + h) + (x + 2h) = 180$ degrees and $x + 2h = 90$ degrees. It follows that $2x + h = 90$ degrees as well.

Therefore, $x = h = 30$ degrees which means the angles of the triangle are 30, 60 and 90 degrees. Now, choose a point M on the hypotenuse BC so that the angle MAC is 60 degrees. That will imply that the angle MAB is 30 degrees, the triangle MAC is equilateral and triangle MAB is isosceles. It follows that the following segments are all equal: $AC = MC = AM = BM$, and therefore M is the midpoint of side BC and $AC = \frac{BC}{2}$.

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17. A 2-digit integer is added to the integer formed by reversing its digits. Which of the following is the greatest factor of that sum?

(a) 1
(b) 2
(c) 7
(d) 11
(e) 13

Correct Answer: (d) 11
Solution: Let $10a + b$ be any 2-digit integer. Then reversing those digits and adding it to the original number produces $10a + b + 10b + a = 11a + 11b$. This number has 11 as a factor.

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18. A parabola in the xy-plane is known to have its vertex at (4, 5) and its focus 3 units below the vertex. What is its equation?

(a) $(x + 4)^2 = 12(y - 5)$
(b) $(x - 5)^2 = -12(y - 4)$
(c) $(x + 4)^2 = -12(y - 5)$
(d) $(x - 5)^2 = 12(y - 4)$
(e) $(x - 4)^2 = -12(y - 5)$

Correct Answer: (e) $(x - 4)^2 = -12(y - 5)$
Solution: The equation of any parabola, with vertex (h, k) that opens downward, is $(x - h)^2 = -4p(y - k)$ where p is the distance between the vertex and the focus. Thus, the answer is $(x - 4)^2 = -12(y - 5)$.

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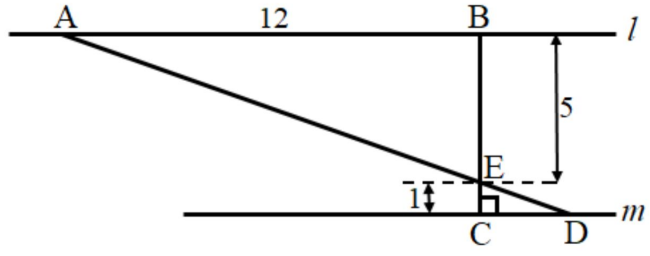
19. In the figure below, line l is parallel to line m, segment BC is perpendicular to line m, and segment AD and segment BC intersect at point E. What is the length of segment AD? If necessary, round to the nearest tenth.



(a) 13.0 units

(b) 13.4 units
(c) 14.6 units
(d) 15.6 units
(e) 20.4 units

Correct Answer: (d) 15.6 units
Solution: Since segment AD and segment BC intersect at E, $\angle AEB$ and $\angle BEC$ are vertical angles, and so the measure of $\angle AEB$ is equal to the measure of $\angle BEC$. Since line l is parallel to line m, $\angle CDE$ and $\angle BAE$ are alternate interior angles of parallel lines cut by a transversal, and so the measure of $\angle CDE$ is equal to the measure of $\angle BAE$. By the angle-angle theorem, $\triangle AEB$ is similar to $\triangle DEC$, with vertices A, E, and B corresponding to vertices D, E, and C, respectively.

Also, $\triangle AEB$ is a right triangle, so by the Pythagorean theorem, $AE = \sqrt{AB^2 + BE^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$. Since $\triangle AEB$ is similar to $\triangle DEC$, the ratios of the lengths of corresponding sides of the two triangles are in the same proportion, which is $\frac{ED}{EB} = \frac{1}{5} = 5$. Thus, $\frac{AE}{EC} = \frac{13}{EC} = 5$, and so $EC = \frac{13}{5}$ or 2.6. Therefore, $AC = AE + EC = 13 + 2.6 = 15.6$.

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20. Ten live bugs move randomly inside a square of side length 1 foot. At any given time, two of the bugs are at a distance less than:

(a) $\frac{1}{2}$ of a foot
(b) $\frac{1}{4}$ of a foot
(c) $\frac{1}{3}$ of a foot
(d) $\frac{1}{6}$ of a foot
(e) $\frac{1}{8}$ of a foot

Correct Answer: (a) $\frac{1}{2}$ of a foot
Solution: Imagine you divide each of the sides of the square into 3 equal parts and then connect the division points by horizontal and vertical lines. You have divided the big square into 9 equal smaller squares. The diagonal of each of these smaller squares is a third of the square root of 2, hence just less than 0.5 ft. or $\frac{1}{2}$ of a foot. There's 10 bugs inside the larger square so at any moment two of them will be in the same smaller square, and therefore at a distance less than the diagonal of the smaller square ($\frac{1}{2}$ of a foot).

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21. What is the sum of all positive divisors of 1648?

(a) 1,576
(b) 3,121
(c) 3,208
(d) 3,224
(e) None of the above

Correct Answer: (d) 3,224.

Solution: The prime factorization of 1,648 is $2^4 \cdot 103$. The factors of $2^4 = 16$ are 1, 2, 4, 8, and 16. The rest of the factors are $1 \cdot 103$, $2 \cdot 103$, …, $16 \cdot 103$. Adding these we get $(1 + 2 + 4 + 8 + 16) \cdot 104 = 31 \cdot 104 = 3{,}224$.

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22. The Great Pyramid in Egypt has a square base measuring 230 meters on each side, and the distance from one corner of the base to the tip of the pyramid is 219 meters. What is the height of the pyramid, rounded to nearest whole number?

(a) 147 meters
(b) 121 meters
(c) 219 meters
(d) 112 meters
(e) None of above

Correct Answer: (a)
Solution: Let x be the distance from corner of base to center of base. Then, $x^2 = 115^2 + 115^2 = 26,450$. The height of pyramid is $h = \sqrt{219^2 - x^2} = \sqrt{47,961 - 26,450} = \sqrt{21,511} \approx \sqrt{21,609} = \sqrt{147^2} = 147$ meters.

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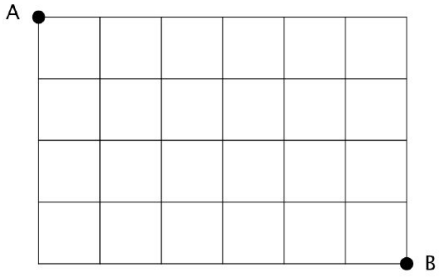
23. A bug wishes to travel from point A to point B on the given diagram. If it can only move down or to the right, how many possible paths are there from A to B?



(a) 60
(b) 130
(c) 180
(d) 210
(e) 270

Correct Answer: (d) 210.
Solution: A valid path consists of a 10-letter sequence built from the letters D (down) and R (right), containing 4 Ds and 6 Rs. For example, the sequence DDDDRRRRRR gives the path that goes from point A immediately to the bottom, and then directly right to point B. The number of such sequences is the number of ways to choose (without order) four objects from 10 given objects. This is $\binom{10}{4} = \frac{10!}{4! \cdot 6!} = 210$.

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24. If the eggs in a basket are removed six at a time, five eggs will remain. If the eggs are removed five at a time, only four will remain. If the eggs are removed four, three, or two at a time, then three, two, and one egg will remain, respectively. But if the eggs are taken out seven at a time, no eggs will be left over. What is the least number of eggs that could be in the basket?

(a) 49
(b) 59
(c) 119
(d) 301
(e) 539

Correct Answer: (c) 119
Solution: We know from the clues given that the number of eggs is one less than a multiple of 2, one less than a multiple of 3, one less than a multiple of 4, one less than a multiple of 5, one less than a multiple of 6, and a multiple of 7. We can represent this by letting N be the number of eggs in the basket and noticing that:

$N = 2A - 1 = 3B - 1 = 4C - 1 = 5D - 1 = 6E - 1 = 7F$ for some integers A, B, C, D, E, F.

This means that $N + 1 = 2A = 3B = 4C = 5D = 6E$. Consequently N must be one less than a multiple of 60 and a multiple of 7. 59 doesn't work, but 119 does.

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25. Consider the points of a plane: $A(2, -3)$ and $B(-2, 3)$. Which of the following is the coordinates of a point C, so the points A, B, and C form the vertices a triangle?

(a) (52, –78)
(b) (3, –4.5)
(c) (–56, 84)
(d) (–100, 150)
(e) None of the above.

Correct Answer: (e) None of the above.
Solution: An equation of the line passing through the points A and B is $y = -\frac{3}{2}(x + 2) + 3$, and every listed ordered pair of numbers satisfies this equation.

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26. Find next number in the sequence 1, 1, 3, 7, 17, ...

(a) 41
(b) 19
(c) 31
(d) 24

Correct Answer: (a) 41.
Solution: The pattern is $a_{n+1} = a_{n-1} + 2a_n$. $7 + 2(17) = 41$.

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27. A snack pack of Starbursts contains two candies. The candies might be pink, yellow, orange, or red. Each color is equally likely to appear. What is the probability of opening a snack pack with at least one yellow candy?

(a) $\frac{1}{4}$
(b) $\frac{3}{16}$
(c) $\frac{3}{8}$
(d) $\frac{2}{5}$
(e) $\frac{7}{16}$

Correct Answer: (e) $\frac{7}{16}$
Solution: If counting, when the order matters, there are 16 possible snack packs. At least one yellow includes the cases of (1) two yellows, (2) first one yellow and second one not yellow, and (3) first one not yellow and second one yellow. The probabilities of these cases are $\frac{1}{16} + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{7}{16}$.

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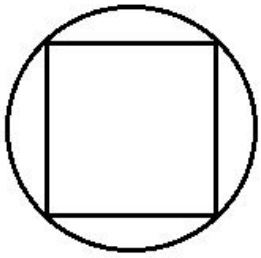
28. A sawmill plans to cut a long cylindrical log into a shape of a beam whose cross-section is a square (see the picture below). What percentage of the wood from the log is being wasted?



(a) 14%
(b) 28%
(c) 36%
(d) 57%
(e) None of the above.

Correct Answer: (c) 36%.
Solution: Let the side of the square be s and the length of the log be l, then the diameter $d = 2r = \sqrt{2}s$, so $r = \frac{\sqrt{2}}{2}s$ and the volume of the log is $V_1 = \frac{1}{2}s^2 l$. The volume of the beam is $V_2 = s^2 l$. So the wasted wood is $V_3 = V_1 - V_2 = \left(\frac{\pi}{2} - 1\right)s^2 l$.

The percentage of the wasted wood from the log is $\frac{V_3}{V_1} = \frac{\pi - 2}{\pi} = 0.36 = 36\%$.

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29. Find the greatest common divisor of the numbers $11^2 \times 13^5 \times 14^2 \times 17^3$ and $6^5 \times 11^8 \times 13^1 \times 19^{18}$.

(a) 6,292
(b) 1,573
(c) 12,012
(d) 286
(e) 143

Correct Answer: (a) 6,292.
Solution: Note that $14 = 2 \times 7$ and $6 = 2 \times 3$. The largest common divisor of two numbers is built from the prime factorizations of the numbers. Specifically, we use the smallest power corresponding to each (common) prime in the factorizations. In this case, we have $2^2 \times 11^2 \times 13^1 = 6{,}292$.

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30. You are selecting from the set of letters {A, B, C, D, E, F}. There are 21 ways you can select two letters with repetition and order is not important. How many ways can you select two letters from a set of 12 different letters with repetition and order is not important?

(a) 78
(b) 42
(c) 29
(d) 252
(e) None of above

Correct Answer: (a) 78
Solution: 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1

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31. John and David would like to purchase a notebook. When John tried to buy the notebook, he was 63 cents short. When David tried to by the same notebook, he was 1 cent short. And when they combined their money and tried to buy the same notebook, they still did not have sufficient funds. How much does the notebook cost?

(a) 62 cents
(b) 63 cents
(c) 64 cents
(d) 65 cents
(e) None of above.

Correct Answer: (b) 63 cents.
Solution: If John had any funds, then combined they would be able to buy the notebook since David was only 1 cent short. Hence, John had no money, and the notebook costs 63 cents.

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32. Vera designs costumes for a local theater company. For an upcoming play she must design a poodle skirt for the lead actress. A poodle skirt is constructed from an entire circle of fabric, with a circle cut out of the center to fit the wearer's waist, and the outer edge of the circle is the bottom edge of the skirt. (See diagram below). Vera plans to trim the bottom edge of the skirt

with horsehair braid to stiffen it. If the actress who will wear the skirt has a 27 inch waist and the skirt must be 30 inches long, how many yards of horsehair braid must Vera buy to be able to trim the edge of the skirt? Round your answer up to the nearest eighth of a yard.

(a) 5 yards
(b) $5\frac{1}{4}$ yards
(c) 6 yards
(d) $7\frac{5}{8}$ yards
(e) $215\frac{1}{2}$ yards

Correct Answer: (c) 6 yards
Solution: Since the actress's waist is 27 inches, the radius of the center hole, r, will be $r = \frac{27}{2\pi}$ inches. The skirt must be 30 inches long, so the radius, R, of the outer circle is $R = 30 + \frac{27}{2\pi}$. Then the circumference of the outer circle is
$C = 2\pi R = 2\pi\left(30 + \frac{27}{2\pi}\right) = 60\pi + 27 \approx 60(3.14) + 27 = 215.4$ inches. Dividing this number by 36 inches gives approximately 5.98 yards. Rounding up to the nearest eighth of a yard gives 6 yards.

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33. Suppose a bag contains the five letters of the word STATE. If you take one letter out at a time and line them up from left to right, what is the probability that you will spell the word STATE?

(a) $\frac{1}{5}$
(b) $\frac{1}{6}$
(c) $\frac{1}{24}$
(d) $\frac{1}{60}$
(e) $\frac{1}{120}$

Correct Answer: (d) $\frac{1}{60}$
Solution: There are two ways to get the letters for the word STATE and there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to arrange five letters. Thus, $\frac{2}{120} = \frac{1}{60}$.

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34. A radio tower sends out a signal in all directions. If the total area of signal coverage is approximately 32,000 square meters, what is the range (in meters) of the linear distance from the radio tower to the outermost edge of the coverage area?

(a) 60 meters to 70 meters
(b) 80 meters to 90 meters
(c) 100 meters to 110 meters
(d) 120 meters to 130 meters
(e) 140 meters to 150 meters

Correct Answer: (c) 100 meters to 110 meters
Solution: The frequency signal from the radio tower creates a circular area of coverage. The linear distance from the tower to the outermost edge of the coverage area is the *radius* of the circular area.

Given that the area of a circle is $A = \pi r^2$, we can compare an estimate of the area without using a calculator to one where you might use a calculator. We will compare calculations using $\pi = 3.14$ and $\pi = 3$.

Way 1 ($\pi = 3.14$):
$A = \pi r^2$
$32{,}000 = (3.14)r^2$
$(32{,}000) \div (3.14) = r^2$
$10{,}191 \approx r^2$

Way 2 ($\pi = 3$):
$A = \pi r^2$
$32{,}000 = (3)r^2$
$(32{,}000) \div (3) = r^2$
$10{,}667 \approx r^2$

Using our possible answers, the estimated ranges are:

(a) 60 meters to 70 meters
Since we know r^2, $(60\text{ m})^2$ to $(70\text{ m})^2 \rightarrow (60)(60)\text{ m}^2$ to $(70)(70)\text{ m}^2 \rightarrow 3600\text{ m}^2$ to 4900 m^2

(b) 80 meters to 90 meters
$(80\text{ m})^2$ to $(90\text{ m})^2 \rightarrow (80)(80)\text{ m}^2$ to $(90)(90)\text{ m}^2 \rightarrow 6400\text{ m}^2$ to 8100 m^2

(c) 100 meters to 110 meters
$(100\text{ m})^2$ to $(110\text{ m})^2 \rightarrow (100)(100)\text{ m}^2$ to $(110)(110)\text{ m}^2 \rightarrow 10{,}000\text{ m}^2$ to $12{,}100\text{ m}^2$

(d) 120 meters to 130 meters
$(120\text{ m})^2$ to $(130\text{ m})^2 \rightarrow (120)(120)\text{ m}^2$ to $(130)(130)\text{ m}^2 \rightarrow 14{,}400\text{ m}^2$ to $16{,}900\text{ m}^2$

(e) 140 meters to 150 meters
$(140\text{ m})^2$ to $(150\text{ m})^2 \rightarrow (140)(140)\text{ m}^2$ to $(150)(150)\text{ m}^2 \rightarrow 1{,}9600\text{ m}^2$ to $22{,}500\text{ m}^2$

The radius squared has a value of $10{,}191\text{ m}^2$ (or $10{,}667\text{ m}^2$ depending on the approximation for π) and the only range which that value falls into is the 100 meters to 110 meters.

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35. This year, a company decreases its advertising budget by *p* percent. A employee's salary is also decreased by *p* percent. What should the percentage of increase be in the next year's advertising budget to bring the employee's salary back to where it was?

(a) $\frac{2p}{100-p}$
(b) $\frac{p-100}{100-2p}$
(c) $\frac{100p}{100-p}$
(d) $\frac{p}{p-100}$
(e) $\frac{2p}{p-100}$

Correct Answer: (c) $\frac{100p}{100-p}$
Solution: Let S be her original salary and x be the percentage needed to bring it back. Then we want $S = \left(S - \frac{p}{100} \cdot S\right) + \frac{x}{100} \cdot \left(S - \frac{p}{100} \cdot S\right)$. This is equivalent to $\frac{100x - xp - 100p}{10{,}000} = 0$, which gives us answer $x = \frac{100p}{100-p}$.

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36. Four customers came into a bakery. The first one said, "Give me half of all the doughnuts you have in your display case, plus half a doughnut more." The second customer said, "Give me half of all the doughnuts you have left in your case, plus half a doughnut more." The third customer said, "Give me three doughnuts." The last customer said, "Give me half of all the doughnuts you have left in your case, plus half a doughnut more." This last transaction emptied the display case of doughnuts. How many doughnuts were there to start with?

(a) 5
(b) 17.5
(c) 18
(d) 19
(e) 21

Correct Answer: (d) 19
Solution: We can find the solution by going backwards through the problem, having the customers put doughnuts in the display case instead of taking them. The last customer puts half of a doughnut in the case and twice that makes 1 doughnut. The third customer puts 3 doughnuts in the case so it now has 4 doughnuts. The second customer puts half a doughnut in the case so it now has $4\frac{1}{2}$ doughnuts and twice that makes 9 doughnuts. The first customer puts half a doughnut in the case so it now has $9\frac{1}{2}$ doughnuts and twice that makes it so the case has 19 doughnuts in the beginning.

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37. If six is subtracted from the largest of three consecutive integers and the difference is doubled, the answer is the same as 20 less than the sum of the smallest integer and twice the middle integer. What is the value of the middle integer in the sequence?

(a) 7
(b) 8
(c) 10
(d) 11
(e) 15

Correct Answer: (d) 11
Solution: Let the smallest integer be x. Then the middle integer will be $(x + 1)$ and the largest integer will be $(x + 2)$. Next, we set up the following equation:

$$\begin{aligned}
&(\text{largest integer} - 6) \cdot 2 = [\text{smallest integer} + 2(\text{middle integer})] - 20 \\
&[(x + 2) - 6] \cdot 2 = [x + 2(x + 1)] - 20 \\
&[x - 4] \cdot 2 = [x + 2x + 2] - 20 \\
&[x - 4] \cdot 2 = [3x + 2] - 20 \\
&2x - 8 = 3x - 18 \\
&10 = x
\end{aligned}$$

So, the smallest integer is 10, the middle integer is 11, and the largest integer is 12.

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38. A captain had 28 troops before a battle. After the battle, if you divide the number of remaining troops by 3, the remainder is 2. If you divide the number of remaining troops by 5, the remainder is 4. How many troops were left after the battle?

(a) 14
(b) 15
(c) 16
(d) 27
(e) None of the above.

Correct Answer: (a) 14.
Solution: The non-negative integers, no greater than 28, which produce the remainder 4 when divided by 5 are: 4, 9, **14**, 19, and 24. And the non-negative integers which produce the remainder 2 when divided by 3 are: 2, 5, 8, 11, **14**, 17, 20, 23, and 26.

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39. What is the maximum area of a right triangle inscribed a circle of radius, r?

(a) r^2
(b) πr^2
(c) $2\pi r^2$
(d) $\frac{1}{2}d^2$
(e) $\frac{1}{2}r^2$

Correct Answer: (a) r^2
Solution: All right triangles inscribed a circle have the diameter of the circle for the hypotenuse. Since the arc of an inscribed angle is twice the angle so if the angle is 90 degrees, the arc is 180 degrees. If we use the diameter as the base of the right triangle, then the triangle with the maximum area must be the triangle with the largest height, since the base is a fixed length (the diameter). The height would be the highest as the radius. So the base is $2r$, the height of the largest triangle is r so the area = $\left(\frac{1}{2}\right)(2r)(r) = r^2$.

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40. How many rectangles exist whose side lengths are whole numbers and their area is 144 square units?

(a) 1
(b) 14
(c) 12
(d) 8
(e) 10

Correct Answer: (d) 8
Solution: $144 = 1 \times 144 = 2 \times 72 = 3 \times 48 = 4 \times 36 = 6 \times 24 = 8 \times 18 = 9 \times 16 = 12 \times 12$.
