Name: _____

School: _____

Grade: _____

2019 State Math Contest (Junior Exam)

Weber State University

March 5, 2019

Instructions:

- Do not turn this page until your proctor tells you.

- Enter your name, grade, and school information following the instructions given by your proctor.

- Calculators are not allowed on this exam.

- This is a multiple choice test with 40 questions. Each question is followed by answers marked (a), (b), (c), (d), and (e). Only one answer is correct.

- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.

- Scoring: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

- You will have 2 hours and 30 minutes to finish the test.

- You may not leave the room until at least 10:15 a.m.

1. Find the product of all real solutions to the equation $x^4 - 10x^2 - 24 = 0$.

    (a) –24
    (b) –12
    (c) –2
    (d) 2
    (e) 12

2. Joe can mop the kitchen floor in 12 minutes. Liz can mop the kitchen floor in 6 minutes. If Joe, Liz, and Zack work together, they can mop the kitchen floor in 2 minutes and 40 seconds. How long does it take Zack to mop the kitchen floor by himself?

    (a) 4 minutes
    (b) 6 minutes
   (c) 8 minutes
    (d) 10 minutes
    (e) 12 minutes

3. The complex number $3 - 2i$ is a root of the polynomial $5x^3 - 32x^2 + 77x - 26$. What is the imaginary part of the other complex root of this polynomial?

    (a) $\frac{2}{3}$
    (b) 1
    (c) $\frac{3}{2}$
    (d) 2
    (e) 3

4. Suppose $x + \frac{1}{y} = 2$ and $y + \frac{1}{x} = 4$. What is the value of $\frac{y}{x}$?

    (a) $\frac{1}{2}$
    (b) 2
    (c) $\frac{1}{4}$
    (d) 4
    (e) 1

5. The ratio of vinegar to water in three separate solutions of vinegar and water is 1:1, 1:2, and 1:3. If equal quantities of each solution are combined together and mixed, what will be the ratio of vinegar to water in the resulting solution?

    (a) 1:6
    (b) 3:6
    (c) 7:12
    (d) 11:18
    (e) 13:23

6. On a "prime day," both the month and the day are prime. For example, March 13th is a prime day, because March is month 3, which is a prime, and so is 13. How many prime days are there in 2019?

(a) 52
(b) 53
(c) 55
(d) 50
(e) None of the above

7. A new car is being developed that uses its entire fuel supply in 38 hours when left idling. However, when driven at 60 mph on a test track, the car uses about $3\frac{1}{2}$ times as much fuel per hour as it does when idling. Suppose the car has been idling for 10 hours, and then it is run at 60 mph on the test track. How much longer will the car run before it uses up all of its fuel?

(a) 28 hours
(b) 10 hours
(c) 8 hours
(d) 7 hours
(e) 3 hours

8. In order to buy a $125,000 house, a couple puts down $25,000 and takes out a mortgage loan on the balance from a bank. Their mortgage payment is $877.57 per month for 360 months. To pay off the mortgage loan, they pay $877.57 per month for the following 360 months. When the mortgage loan is fully paid off, how much more will they end up paying for the house than the original price? (Round you answer to the nearest whole number.)

(a) $215,925
(b) $216,080
(c) $315,925
(d) $340,925
(e) $341,080

9. What is the remainder when 2^{1000} is divided by 13?

(a) 2
(b) 3
(c) 4
(d) 10
(e) 12

10. You are driving to a job interview and do not want to be late. You figure you must average 60 miles per hour for the remaining two miles to the interview to be on time. Due to an accident, you average 30 miles per hour for the first mile. What speed do you need to average for the second mile to make it to your interview on time?

(a) 90 miles per hour
(b) 30 miles per hour
(c) 120 miles per hour
(d) 105 miles per hour
(e) It is impossible to be on time to the interview.

11. How many positive integers less than 50 have an odd number of different factors.

(a) 7
(b) 11
(c) 13
(d) 17
(e) None of the above

12. If three fair dice are rolled, what is the probability that the sum of the three dice is 6?

(a) $\frac{5}{108}$
(b) $\frac{1}{18}$
(c) $\frac{1}{24}$
(d) $\frac{10}{36}$
(e) $\frac{1}{27}$

13. The largest circle pictured has a radius $r = 49$ units. What is the area of the shaded region? (For simplicity, use $\pi = \frac{22}{7}$.)
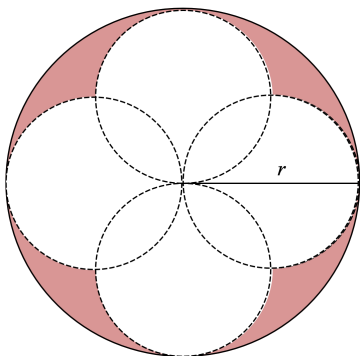
(a) 2401 units squared
(b) 1372 units squared
(c) 2744 units squared
(d) 7546 units squared
(e) 5488 units squared

14. Which of the following numbers is a factor of 13,580,237?

    (a) 3
    (b) 6
    (c) 9
    (d) 7
    (e) 11

15. Write 0.224 in base 5.

    (a) 0.110_5
    (b) 0.103_5
    (c) 0.224_5
    (d) 0.112_5

16. If the angles of a right triangle are in *arithmetic progression* then the ratio of the smallest side is to the largest side is:

    (a) $\frac{1}{3}$
    (b) $\frac{1}{4}$
    (c) $\frac{1}{2}$
    (d) $\frac{2}{3}$

17. A 2-digit integer is added to the integer formed by reversing its digits. Which of the following is the greatest factor of that sum?

    (a) 1
    (b) 2
    (c) 7
    (d) 11
    (e) 13

18. A parabola in the xy-plane is known to have its vertex at $(4, 5)$ and its focus 3 units below the vertex. What is its equation?

    (a) $(x + 4)^2 = 12(y - 5)$
    (b) $(x - 5)^2 = -12(y - 4)$
    (c) $(x + 4)^2 = -12(y - 5)$
    (d) $(x - 5)^2 = 12(y - 4)$
    (e) $(x - 4)^2 = -12(y - 5)$

19. In the figure below, line l is parallel to line m, $\overline{BC}$ is perpendicular to line m, and $\overline{AD}$ and $\overline{BC}$ intersect at point E. What is the length of $\overline{AD}$? If necessary, round to the nearest tenth.

A   12   B l

5

E

1

C   D m
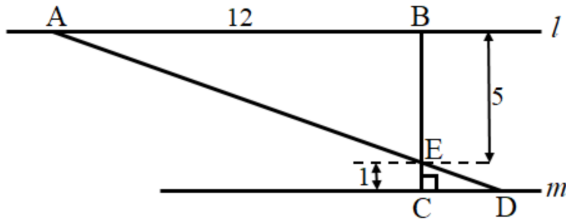
(a) 13.0 units
(b) 13.4 units
(c) 14.6 units
(d) 15.6 units
(e) 20.4 units

20. Ten live bugs move randomly inside a square of side length 1 foot. At any given time, two of the bugs are at a distance less than:

(a) $\frac{1}{2}$ of a foot
(b) $\frac{1}{4}$ of a foot
(c) $\frac{1}{3}$ of a foot
(d) $\frac{1}{6}$ of a foot
(e) $\frac{1}{8}$ of a foot

21. What is the sum of all positive divisors of 1,648?

(a) 1,576
(b) 3,121
(c) 3,208
(d) 3,224
(e) None of the above

22. The Great Pyramid in Egypt has a square base measuring 230 meters on each side, and the distance from one corner of the base to the tip of the pyramid is 219 meters. What is the height of the pyramid, rounded to nearest whole number?

(a) 147 meters
(b) 121 meters
(c) 219 meters
(d) 112 meters
(e) None of above

23. A bug wishes to travel from point A to point B on the given diagram. If it can only move down or to the right, how many possible paths are there from point A to point B?


A ... B
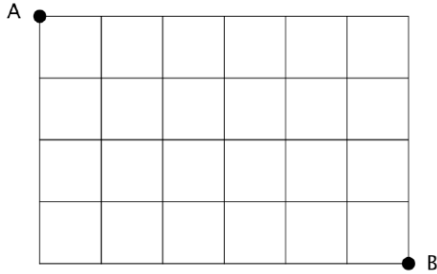
(a) 60
(b) 130
(c) 180
(d) 210
(e) 270

24. If the eggs in a basket are removed 6 at a time, 5 eggs will remain. If the eggs are removed 5 at a time, only 4 will remain. If the eggs are removed 4, 3, or 2 at a time, then 3, 2, and 1 egg will remain, respectively. But if the eggs are taken out 7 at a time, no eggs will be left over. What is the least number of eggs that could be in the basket?

(a) 49
(b) 59
(c) 119
(d) 301
(e) 539

25. Consider the points of a plane: $A(2, -3)$ and $B(-2, 3)$. Which of the following is the coordinates of a point C, so the points A, B, and C form the vertices a triangle?

(a) (52, –78)
(b) (3, –4.5)
(c) (–56, 84)
(d) (–100, 150)
(e) None of the above

26. Find next number in the sequence 1, 1, 3, 7, 17, ...

(a) 41
(b) 19
(c) 31
(d) 24

27. A snack pack of Starbursts contains two candies. The candies might be pink, yellow, orange, or red. Each color is equally likely to appear. What is the probability of opening a snack pack with at least one yellow candy?

    (a) $\frac{1}{4}$
    (b) $\frac{3}{16}$
    (c) $\frac{3}{8}$
    (d) $\frac{2}{5}$
    (e) $\frac{7}{16}$

28. A sawmill plans to cut a long cylindrical log into a shape of a beam whose cross-section is a square (see the picture below). What percentage of the wood from the log is being wasted?
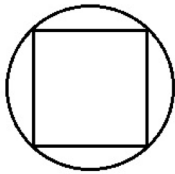
    (a) 14%
    (b) 28%
    (c) 36%
    (d) 57%
    (e) None of the above

29. Find the greatest common divisor of the numbers $11^2 \times 13^5 \times 14^2 \times 17^3$ and $6^5 \times 11^8 \times 13^1 \times 19^{18}$.

    (a) 6,292
    (b) 1,573
    (c) 12,012
    (d) 286
    (e) 143

30. You are selecting from the set of letters {A, B, C, D, E, F}. There are 21 ways you can select two letters with repetition and order is not important. How many ways can you select two letters from a set of 12 different letters with repetition and order is not important?

    (a) 78
    (b) 42
    (c) 29
    (d) 252
    (e) None of above

31. John and David would like to purchase a notebook. When John tried to buy the notebook, he was 63 cents short. When David tried to by the same notebook, he was 1 cent short. And when they combined their money and tried to buy the same notebook, they still did not have sufficient funds. How much does the notebook cost?

(a) 62 cents
(b) 63 cents
(c) 64 cents
(d) 65 cents
(e) None of above

32. Vera designs costumes for a local theater company. For an upcoming play she must design a poodle skirt for the lead actress. A poodle skirt is constructed from an entire circle of fabric, with a circle cut out of the center to fit the wearer's waist, and the outer edge of the circle is the bottom edge of the skirt. (See diagram below). Vera plans to trim the bottom edge of the skirt with horsehair braid to stiffen it. If the actress who will wear the skirt has a 27 inch waist and the skirt must be 30 inches long, how many yards of horsehair braid must Vera buy to be able to trim the edge of the skirt? Round your answer up to the nearest eighth of a yard.

(a) 5 yards
(b) $5\frac{1}{4}$ yards
(c) 6 yards
(d) $7\frac{5}{8}$ yards
(e) $215\frac{1}{2}$ yards

33. Suppose a bag contains the five letters of the word STATE. If you take one letter out at a time and line them up from left to right, what is the probability that you will spell the word STATE?

(a) $\frac{1}{5}$
(b) $\frac{1}{6}$
(c) $\frac{1}{24}$
(d) $\frac{1}{60}$
(e) $\frac{1}{120}$

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34. A radio tower sends out a signal in all directions. If the total area of signal coverage is approximately 32,000 square meters, what is the range (in meters) of the linear distance from the radio tower to the outermost edge of the coverage area?

   (a) 60 meters to 70 meters
   (b) 80 meters to 90 meters
   (c) 100 meters to 110 meters
   (d) 120 meters to 130 meters
   (e) 140 meters to 150 meters

35. This year, a company decreases its advertising budget by p percent. A employee's salary is also decreased by p percent. What should the percentage of increase be in the next year's advertising budget to bring the employee's salary back to where it was?

   (a) $\frac{2p}{100-p}$
   (b) $\frac{p-100}{100-2p}$
   (c) $\frac{100p}{100-p}$
   (d) $\frac{p}{p-100}$
   (e) $\frac{2p}{p-100}$

36. Four customers came into a bakery. The first one said, "Give me half of all the doughnuts you have in your display case, plus half a doughnut more." The second customer said, "Give me half of all the doughnuts you have left in your case, plus half a doughnut more." The third customer said, "Give me three doughnuts." The last customer said, "Give me half of all the doughnuts you have left in your case, plus half a doughnut more." This last transaction emptied the display case of doughnuts. How many doughnuts were there to start with?

   (a) 5
   (b) 17.5
   (c) 18
   (d) 19
   (e) 21

37. If six is subtracted from the largest of three consecutive integers and the difference is doubled, the answer is the same as 20 less than the sum of the smallest integer and twice the middle integer. What is the value of the middle integer in the sequence?

   (a) 7
   (b) 8
   (c) 10
   (d) 11
   (e) 15

38. A captain had 28 troops before a battle. After the battle, if you divide the number of remaining troops by 3, the remainder is 2. If you divide the number of remaining troops by 5, the remainder is 4. How many troops were left after the battle?

(a) 14
(b) 15
(c) 16
(d) 27
(e) None of the above

39. What is the maximum area of a right triangle inscribed a circle of radius, r?

(a) r^2
(b) πr^2
(c) $2\pi r^2$
(d) $\frac{1}{2}d^2$
(e) $\frac{1}{2}r^2$

40. How many rectangles exist whose side lengths are whole numbers and their area is 144 square units?

(a) 1
(b) 14
(c) 12
(d) 8
(e) 10