Name:_____

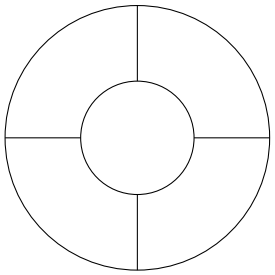
Student ID:_____

State Math Contest (Junior)

Instructions:

- Do not turn this page until your proctor tells you.
- Enter your name, grade, and school information following the instructions given by your proctor.
- Calculators are **not** allowed on this exam.
- This is a multiple choice test with 40 questions. Each question is followed by answers marked a), b), c), d), and e). Only one answer is correct.
- Mark your answer to each problem on the bubble sheet Answer Form with a #2 pencil. Erase errors and stray marks. Only answers properly marked on the bubble sheet will be graded.
- **Scoring:** You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
- You will have 2 hours and 30 minutes to finish the test.

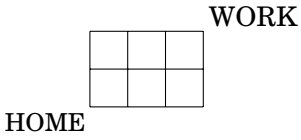
1. In the figure below we want to color the five regions. If we have to color the regions so that it does not have the same color as an adjacent region, what is the fewest number of colors we can use?



a) 1 b) 2 c) 3

d) 4 e) 5

2. A young man has to walk from his home to his work. (His mom picks him up so he does not walk home.) How many days can he walk to work in a different way by walking a total of 2 blocks north and 3 blocks east in any order? (No cutting diagonal through the blocks).



a) 1 b) 5 c) 8

d) 10 e) 12

3. The current in a stream moves at a constant speed. A duck swims 8 miles downstream in 2 hours. She returns upstream in 14 hours. How fast does the duck swim in still water? (You should assume that the duck swims the same speed relative to the water both directions.)

a) 15/7 miles per hour. b) 14/7 miles per hour. c) 16/7 miles per hour.

d) 18/7 miles per hour. e) 13/7 miles per hour.

4. You are rolling 2 dice. What is the probability that the absolute value of the difference of the outcomes is at least 4?

a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{1}{5}$

d) $\frac{1}{9}$ e) $\frac{3}{5}$

5. A triangle in the xy-plane has vertices at $(7,3)$, $(12,3)$, and $(c+7,15)$. Find values of c so that this is a right triangle.

a) 4 b) 3 and -2 c) 8 and -4

d) 0 and -3 e) 0 and 5 f) 9

6. A parabola in the xy-plane is known to have its vertex at $(2,5)$ and its focus 2 units to the left of the vertex. What is its equation?

a) $(y-2)^2 = -8(x-5)$ b) $(y-5)^2 = -8(x-2)$ c) $(y-5)^2 = 2(x-2)$

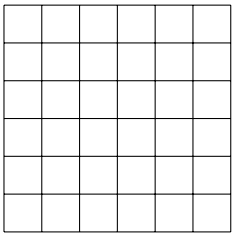
d) $(y-2)^2 = -2(x-5)$ e) $(y-5) = 4(x-2)^2$

7. A bumble bee is traveling back and forth between the front end of two trains moving towards each other. If the trains start 90 miles away from each other and one train is going 10 miles per hour while the other train is going 20 miles an hour and the bumble bee is traveling 100 miles per hour, how many miles does the bumble bee travel before being smashed by the two trains colliding?

a) 450 miles b) 90 miles c) 300 miles

d) 180 miles e) 270 miles

8. How many ways are there to place 6 circles in 6 different squares on the board below so that no circle is in the same row or column as another circle?



a) 360 b) 720 c) 120

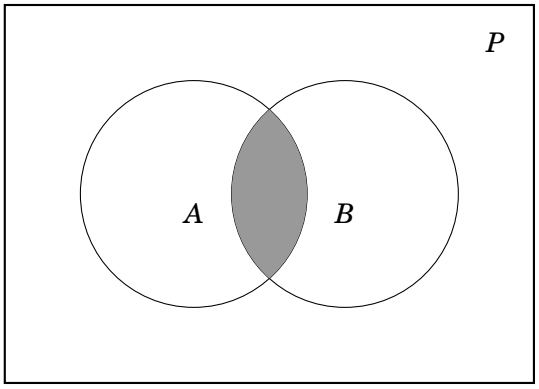
d) 12 e) 36

9. Find the area of triangle ΔABC if $AB = AC = 50$ in and $BC = 60$ in.

a) 2000 square inches b) 1500 square inches c) 1000 square inches

d) 2400 square inches e) 1200 square inches

10. Which of the following expressions indicate the shaded region. ($\overline{A}$ represents the complement of A relative to P.)



I. $A \cup B$ II. $A \cap B$ III. $P \cup (A \cup B)$ IV. $P \cap (A \cup B)$
V. $P \cap (A \cap B)$ VI. $P \cup (A \cap B)$ VII. $\overline{\overline{A} \cup \overline{B}}$ VIII. $\overline{\overline{A} \cap \overline{B}}$

a) only II. b) only V. c) only VIII.

d) II., V., and VII. e) III., V., and VIII.

11. In the sequence of numbers $1, 4, 3, -1, \cdots$ each term after the first two is equal to the term preceding it minus the term preceding that. Find the sum of the first one hundred terms of the sequence.

a) 1 b) -2 c) -3

d) 7 e) 3

12. What is the smallest positive integer that is both a perfect power of 11 ($11^1, 11^2, 11^3 \ldots$) and **not** a palindrome? (A palindrome is a number that reads the same backwards as forwards such as 121 or 1331)

a) 1771561 b) 161151 c) 14631

d) 161051 e) 122

13. A positive whole number leaves a remainder of 7 when divided by 11 and a remainder of 10 when divided by 12. What is the remainder when divided by 66?

a) 0 b) 40 c) 28

d) 52 e) 29

14. Suppose M, N, O, P are real numbers. Suppose that the following two conditions hold.

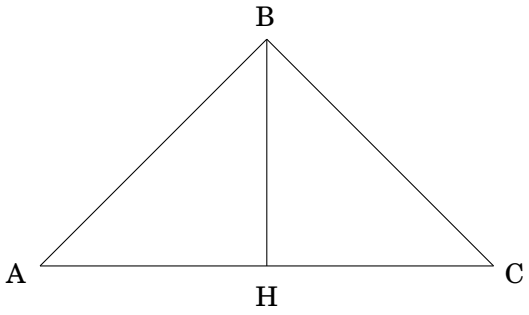
- M is greater than both N and O
- N is greater than O and less than P

Which of the following statements must be true?

a) M is not greater than either O and P. b) O is greater than N. c) P is greater than O.

d) O is greater than P. e) M is greater than P, O, and N.

15. Line BH is perpendicular to line AC. Angle BAC is equal to BCA with both measure $\frac{\pi}{6}$. If BH is length 4, what is the length of AC?
Diagram is may not be drawn to scale.



a) $8\sqrt{3}$ b) $4\frac{\sqrt{3}}{2}$ c) 16

d) 10 e) $4\frac{\sqrt{2}}{2}$

16. Find the sum of all the even integers from 546 to 854 inclusive:

$$546+548+550+\cdots+852+854.$$

a) 106,400 b) 109,200 c) 107,800

d) 107,100 e) 108,500

17. Find a real number a such that equation $||x-a|-a|=2$ has exactly three different solutions.

a) -1 b) 1 c) 0

d) -2 e) 2

18. Let x and y be positive numbers satisfying

$$2+\log_2 x = 3+\log_3 y = \log_6(x+y).$$

Find the value of $\frac{1}{x}+\frac{1}{y}$.

a) 54 b) 36 c) 108

d) 216 e) 81

19. What is the minimal value of $4(x^2+y^2+z^2)-(x+y+z)^2$ when x,y,z are different integers?

a) 8 b) 9 c) 7

d) 6 e) 10

20. The probability that a baseball player gets a hit is 1/5. Find the probability that the player gets exactly 2 hits when batting 4 times in his next game.

a) $\frac{95}{625}$ b) $\frac{98}{625}$ c) $\frac{92}{625}$

d) $\frac{94}{625}$ e) $\frac{96}{625}$

21. Find x if $3^{27^x} = 27^{3^x}$.

a) $x = 1/3$ b) $x = 1/2$ c) $x = 3/2$

d) $x = 1/4$ e) $x = 2/3$

22. Find y if $(2, y)$ lies on the line joining $(0, 3/2)$ and $(9/4, 0)$.

a) $y = -1/6$ b) $y = 1/6$ c) $y = -1/3$

d) $y = 1/3$ e) $y = 5/6$

23. Find the *shortest* path which starts at the origin and visits all five of the following points and returns to the origin: $\{(0,0), (1,0.5), (2,1), (2,0), (0,3)\}$.

a) $(0,0),(1,0.5),(2,1),(2,0),(0,3),(0,0)$ b) $(0,0),(2,0),(1,0.5),(2,1),(0,3),(0,0)$

c) $(0,0),(1,0.5),(2,0),(2,1),(0,3),(0,0)$ d) $(0,0),(0,3),(1,0.5),(2,0),(2,1),(0,0)$

e) $(0,0),(0,3),(2,0),(1,0.5),(2,1),(0,0)$

24. Find the volume of a cone whose base has an area of one square-unit, and whose vertex is one unit above the plane of the base.

a) 1/4 b) 1/3 c) 1/2

d) 2/3 e) 1

25. Find the greatest common divisor of 123432123432100 and 2468642468642000015.

a) 1 b) 3 c) 5

d) 7 e) 15

26. Suppose that $f(x)$ is a polynomial such that $f(x+3) = x^2 + 4x$. What is the sum of the zeros of $f(x)$.

a) 5 b) 2 c) -2

d) 4 e) -3

27. A rectangle is partitioned into 4 subrectangles as shown below. If the subrectangles have the indicated areas, find the area of the unknown rectangle.

60	100
?	80

a) 72 b) 21 c) 70

d) 48 e) 64

28. How many pairs of integers (x, y) satisfy the following equations.

$$x^3 + 6x^2 + 8x = 3y^2 + 9y + 1.$$

a) 0 b) 1 c) 2

d) 3 e) None of the above.

29. A bucket contains 8 red marbles and 4 blue marbles. If we select 3 marbles randomly from the bucket without replacement, what is the probability that no blue marbles are selected?

a) $\frac{7}{36}$ b) $\frac{14}{55}$ c) $\frac{7}{55}$

d) $\frac{14}{36}$ e) None of the above.

30. Two 8.5-inch by 11-inch sheets of paper are laying on a 3 foot by 3 foot table. 1173 square inches of the table is not covered by paper. What is the area of the overlap between the two sheets of paper?

a) 64 sq inches b) 36 sq inches c) 187 sq inches

d) 32 sq inches e) 123 sq inches

31. A bag contains 12 coins consisting of quarters, dimes, and nickels. If the total value of the coins in the bag is at most \$2.15. What is the maximal number of quarters the bag can contain?

a) 7 b) 8 c) 5

d) 6 e) 9

32. A group of airplanes is based on a small island. The tank of each plane holds just enough fuel to take it halfway around the world. Any desired amount of fuel can be transferred from the tank of one plane to the tank of another while the planes are in flight. The only source of fuel is on the island. It is assumed that there is no time lost in refueling either in the air or on the ground. The planes have the same constant speed and rate of fuel consumption. What is the smallest number of planes that will ensure the flight of one plane around the world on a great circle and have all the planes return safely to their island base?

a) 3 b) 4 c) 6

d) 7 e) 9

33. Amy likes either red or green clothes, but not both. She likes either turtleneck or V-neck sweaters, but not both. Amy never wears a sweater that is both the color and the type she likes, nor does she wear one that is neither the color nor the type she likes. Amy wears a red turtleneck. If you want to buy a sweater for Amy that she will wear. Should you buy a red V-neck, a green V-neck, or a green turtleneck?

a) A red V-neck
b) A green V-neck
c) A green turtle neck
d) Amy won't wear any of those sweaters
e) Not enough information is given to answer the question

34. Find the units digit of 13^{2017}.

a) 5
b) 7
c) 9
d) 1
e) 3

35. Find all real numbers a such that $f(x) = x^2 + a|x-1|$ is an increasing function on the interval $[0,\infty)$.

a) $[-2,\infty)$
b) $[-2,0]$
c) $(-\infty,0]$
d) $[0,2]$
e) $[-2,2]$

36. Let $f(x)$ be an odd function defined on $\mathbb{R}$ satisfying

(a) $f(x+2) = -f(x)$, for all real numbers x;
(b) $f(x) = 2x$ when $0 \le x \le 1$.

Find the value of $f(10\sqrt{3})$.

a) $20\sqrt{3} - 36$
b) $-20\sqrt{3} + 36$
c) $-10\sqrt{3} + 18$
d) $10\sqrt{3} + 18$
e) $-20\sqrt{3} - 36$

37. Assume x and y are real numbers and satisfy $x^3 + 2x^2y - 3y^3 = 0$. Then $x^2 + y^2$ must be equal to which of the following?

a) 2
b) $2x$
c) $2x^2$
d) $2x^3$
e) $2y$

38. $\sqrt[3]{2\sqrt{13}+5} - \sqrt[3]{2\sqrt{13}-5} =$

a) -1
b) $2\sqrt{13}$
c) 10
d) 1
e) 3

39. Suppose real numbers x, y, z satisfy the equation $\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} = 1$.

Compute the value of

$$\frac{x^2}{y+z} + \frac{y^2}{z+x} + \frac{z^2}{x+y}.$$

a) 1 b) -1 c) 2

d) 0 e) -2

40. Find the area of overlap between the two circular discs, $x^2 + y^2 = 1$ and $(x-2)^2 + y^2 = 3$.

a) $\frac{\pi}{2} - 1$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2} - \frac{\sqrt{3}}{2}$

d) $\frac{4}{5}$ e) $\frac{5\pi}{6} - \sqrt{3}$.

END OF EXAM