

Utah State Mathematics Contest
Junior Exam
March 18, 2009

1. A metal cylinder is melted down and reshaped into a new cylinder. If the new diameter has been decreased by 20% without changing the volume, then the height has been increased by what percent?
(a) 20 (b) 25 (c) 36 (d) 46.75 (e) 56.25

2. John has coins that may include pennies, nickels, dimes, or quarters. The mean value of the coins is 20 cents. If he were to add one quarter to his money, the new mean value would be 21 cents. How many quarters did he originally have?
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

3. If the quadratic equation $x^2 - px - q = 0$ has two distinct real roots, then:
(a) $p^2 - 4q \geq 0$ (b) $-p^2 - 4q > 0$ (c) $p^2 + 4q \geq 0$
(d) $p^2 > 4q$ (e) $p^2 > -4q$

4. What is the fewest number of people you would need to have in a group if you needed to be certain that at least three of them have the same birth month?
(a) 25 (b) 81 (c) 39 (d) 27 (e) 36

5. When Mary is 64 years old, her age is both a perfect square and a perfect cube. How many more years would she have to live until the next time her age would be both a perfect square and a perfect cube?
(a) 61 (b) 448 (c) 152 (d) 665 (e) 17

6. In the November 2008 general election, Utah voters turned out in strong numbers. Between Utah and Salt Lake counties, 67% of the 840 thousand registered voters made it to the election booth. Of registered voters, 65% of those in Salt Lake County voted, while 72% of those in Utah County voted. How many thousands of registered voters were in Utah County?
(a) 230 (b) 240 (c) 250 (d) 260 (e) 270

7. A digit is placed in each empty square in the grid so that each row and each column contains the digits 1, 2, 3, 4 and 5. What digit must be placed in the square marked 'X'?

	5	4		
1	3			
		5	3	
2		3	1	
				X

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
8. A man either was or will be N years old on his birthday in the year N^2 . Assuming that the man was born in the 20th century, what is the year that he was born?
- (a) 1960 (b) 1980 (c) 1984 (d) 1992 (e) 2000
9. Ninety-six increased by 25% is the same as what number decreased by 25%?
- (a) 160 (b) 120 (c) 192 (d) 144 (e) 200
10. Which of the following has the largest prime factor?
- (a) 121 (b) 51 (c) 91 (d) 87 (e) 133
11. When Greg swims out from the beach, he is carried by the tide; it takes him 4 minutes to reach the nearest buoy. When he swims back in, he takes 16 minutes to swim against the tide. If Greg swims 100 yards per minute in still water, how many yards away is the buoy?
- (a) 400 (b) 480 (c) 640 (d) 720 (e) 800
12. In an online game, you can choose from as many as four characters to be a part of your team. Assuming that, aside from yourself, you include at least one team member, how many different teams can you assemble from the available team members?
- (a) 16 (b) 32 (c) 5 (d) 30 (e) 15
13. Which of the following plane figures is not necessarily convex?
- (a) a regular hexagon (b) a parallelogram (c) a pentagon
(d) a scalene triangle (e) an ellipse

14. Ten acres of grass feeds twenty cows for sixty weeks. How many weeks would fifteen acres feed forty-five cows?

- (a) 30 (b) 40 (c) 45 (d) 52.5 (e) 75

15. The point (6, 2) lies on a line with slope equal to $\frac{2}{5}$. What is the distance between (6, 2) and the closest point on the line with two integer coordinates?

- (a) $\sqrt{29}$ (b) $\sqrt{53}$ (c) $2\sqrt{10}$ (d) $\sqrt{21}$ (e) $\sqrt{49}$

16. Determine the sum of all solutions to the following equation:

$$\sqrt{x} + \sqrt{9-x} = 3$$

- (a) 1 (b) 9 (c) 17 (d) 25 (e) 5

17. Solve the following inequality:

$$\frac{x-6}{x-3} \geq 2$$

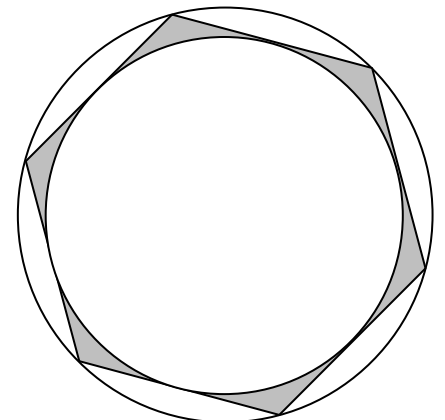
- (a) $\{x \mid x \leq 0\}$ (b) $\{x \mid 6 \geq x > 3\}$ (c) $\{x \mid x \geq 6 \text{ or } 3 < x\}$
 (d) $\{x \mid 3 > x \geq 0\}$ (e) $\{x \mid x > 3 \text{ or } 0 \leq x\}$

18. In the entire 7th grade class at Harbor Middle School, there are 85 students who take algebra and 92 who take American History; some of those mentioned are in both classes. There are 47 students who are enrolled in exactly one of the two. How many students are taking algebra, but not American History?

- (a) 42 (b) 54 (c) 38 (d) 26 (e) 20

19. The picture to the right illustrates a circle inscribed in a regular hexagon inscribed in another circle. What is the ratio of the combined areas of the shaded regions to the combined areas of the unshaded regions?

- (a) $\frac{6\sqrt{3} - 3\pi}{7\pi - 6\sqrt{3}}$ (b) $\frac{4\pi - 6\sqrt{3}}{12\sqrt{3} - \pi}$ (c) $\frac{6\sqrt{3} - 3\pi}{6\pi - 4\sqrt{3}}$
 (d) $\frac{6\sqrt{3} - 3\pi}{4\pi - 6\sqrt{3}}$ (e) $\frac{1}{12}$



20. Jay is comparing fruit at a bazaar. He finds that his thirty-five dollars will evenly buy:
 (1) nineteen pounds of bananas and five pounds of coconuts
 (2) six pounds of bananas and twenty pounds of coconuts
 How much more money would Jay need to buy fifteen pounds of each fruit?

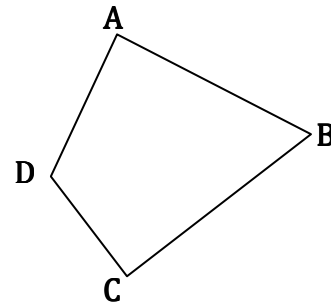
(a) \$4.30 (b) \$2.50 (c) \$10.60 (d) \$7.00 (e) He has enough money

21. How many miles may a person ride on a bike, going at the rate of 8 miles per hour, if he is to be gone 11 hours, and if he must walk back at a rate of 3 miles per hour?

(a) 12 (b) 18 (c) 20 (d) 22 (e) 24

22. Given quadrilateral ABCD with $AB = 11$, $BC = 12$, $CD = 5$, $m\angle A = 90^\circ$, and $m\angle C = 90^\circ$, find the length of side AD.

(a) 6 (b) $4\sqrt{3}$ (c) $7\sqrt{2}$
 (d) $3\sqrt{5}$ (e) $2\sqrt{10}$



23. Which of the following statements are true for all real values of x ?

(I) $|x| \cdot |-x| = x^2$ (II) $|-x| = x$ (III) $|x^3| - |x^2| \geq 0$

(a) I (b) II (c) III (d) I & II (e) I, II & III

24. At 12:00 PM, the second, minute and hour hand on a clock all point to XII. Assuming continuous motion of all three hands, how many times will the second hand pass the minute hand in the time it takes for the minute hand to pass the hour hand three times?

(a) 192 (b) 193 (c) 194 (d) 195 (e) 196

25. Determine which of the following expressions is the least quantity:

(a) $2^{(2^{2^{2^2}})}$ (b) $(2^2)^{(2^{2^2})}$ (c) $(2^{2^2})^{(2^2)}$ (d) $(2^{2^{2^2}})^2$ (e) All are the same amount

26. A fox pursued by a hound has a head-start of 80 of her leaps. The fox makes five leaps while the hound makes only three; however, five hound leaps are equal to nine fox leaps. How many leaps will the hound take before he catches the fox?

- (a) 1000 (b) 880 (c) 1200 (d) 1080 (e) 600

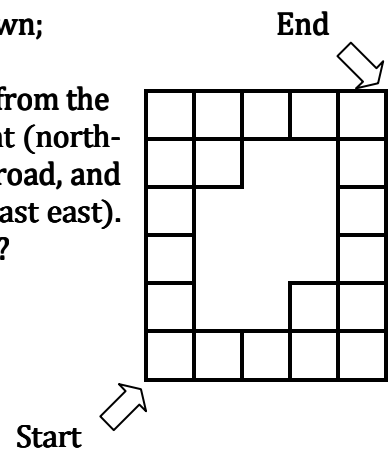
27. If there are two real numbers, X and Y , such that $X < -Y$, then $\sqrt{(X + Y)^2}$ is equivalent to:

- (a) $X + Y$ (b) $-X - Y$ (c) $X - Y$ (d) $Y - X$ (e) none of these

28. Sarah has achieved times of 86, 94, 97, 88, and 96 for swimming 100 meters on her last five attempts. How many seconds would she need on her sixth try to bring her median time down to 92 seconds?

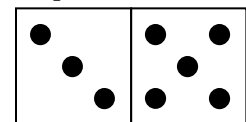
- (a) 92 (b) 87 (c) 91 (d) 90 (e) 89

29. The grid to the right represents a road map of a very small town; the black lines are roads while the spaces represent city blocks, with a large park in the middle. You wish to travel from the bottom left (south-west) corner of the map to the top right (north-east) corner. Assume that you must travel exclusively by road, and that you must always travel up or to the right (north or east). By how many different paths can you accomplish this goal?

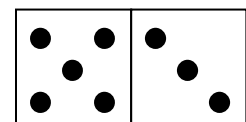


- (a) 30 (b) 156 (c) 102
(d) 120 (e) 84

30. The game of dominoes is made up of rectangular game pieces, each of which has on its upper surface two squares, each containing from 0 to 12 dots. We can represent the domino with three dots in one square and five dots in the other as both the ordered pair $(5, 3)$ and $(3, 5)$ if it is rotated 180° as shown. These two would be considered to be the same domino. Find the maximum number of distinct dominoes which can be created.



- (a) 169 (b) 98 (c) 144 (d) 78 (e) 91



31. The product of $\sqrt[4]{25}$ and $\sqrt[5]{125}$ is equal to:

- (a) $25 \sqrt[9]{5}$ (b) $5 \sqrt[9]{5}$ (c) $5 \sqrt[20]{125}$ (d) $5 \sqrt[10]{5}$ (e) $25 \sqrt[10]{5}$

32. In decimal notation (base 10), the number two-thousand and nine has how many digits in binary notation (base 2)?

- (a) 10 (b) 11 (c) 12 (d) 13 (e) More than 13

33. Assume that you have 12 poles, each measuring a different whole number of meters in length and ranging from 1 meter to 12 meters in length. How many distinct combinations of 3 poles can join end to end to form a triangle?

- (a) 95 (b) 72 (c) 87 (d) 154 (e) 100

34. Determine which factor evenly divides the following polynomial expression:

$$x^3 - 2x^2 - 9x + 60$$

- (a) $x + 2$ (b) $x - 3$ (c) $x + 4$ (d) $x - 5$ (e) $x + 6$

35. You have two marble bags – one leather and the other cotton. The leather bag contains three red and three black marbles. The cotton bag contains six red marbles. You randomly select one marble from each bag and exchange them. If you were to now select a new marble from the leather bag, the probability that it would be red is:

- (a) $\frac{11}{18}$ (b) $\frac{2}{3}$ (c) $\frac{19}{36}$ (d) $\frac{1}{2}$ (e) $\frac{7}{12}$

36. The following “proof” showing that $+1 = -1$ has a mis-statement in which step?

$$+1 \stackrel{\textcircled{1}}{=} \sqrt{1^2} \stackrel{\textcircled{2}}{=} \sqrt{(-1)^2} \stackrel{\textcircled{3}}{=} \sqrt{(-1) \cdot (-1)} \stackrel{\textcircled{4}}{=} \sqrt{(-1)} \cdot \sqrt{(-1)} \stackrel{\textcircled{5}}{=} i \cdot i \stackrel{\textcircled{6}}{=} i^2 \stackrel{\textcircled{7}}{=} -1$$

- (a) 2 (b) 3 (c) 4 (d) 5 (e) 7

37. If $2^x = 15$, and if $15^y = 32$, then what is the value of xy ?

- (a) $\frac{9}{2}$ (b) 5 (c) $\frac{16}{3}$ (d) 6 (e) $\frac{13}{2}$

38. Find the area of the smallest region bounded by the graphs of the equations:

$$y = |x| \quad \text{and} \quad x^2 + y^2 = 20$$

- (a) 5π (b) 25π (c) 20π (d) 4π (e) $\frac{25}{16}\pi$

39. Simplify the expression:

$$\frac{35! + 34!}{35! - 34!}$$

- (a) $\frac{18}{17}$ (b) 2346 (c) $\frac{15}{13}$ (d) 69 (e) $\frac{7}{5}$

40. Five women are playing a card game in which they each have five cards. Each card is one of five different colors. All of the following statements are true:

- (I) Any player who has a yellow card also has an orange card.
 - (II) Only if a player has a yellow and a red card does she have a blue card.
 - (III) A player has a green card if she does not have a yellow card.
 - (IV) A player does not have a blue card only if she does not have an orange card.
 - (V) Of blue, green, yellow and orange cards, each player has at least two colors.
- Only one player currently has one card of each color. This woman is the only player holding a card that is:

- (a) Red (b) Orange (c) Yellow (d) Green (e) Blue