State Senior Mathematics Contest
Spring 2007

1. What is the greatest divisor of 19! and 19! + 17?

   (a) 1 (b) 17 (c) 19 (d) 19! (e) 17!

2. The decimal $0.\overline{9} = 0.999\ldots$ is equal to

   (a) 1 (b) $1-(\frac{9}{10})^{10}$ (c) $(\frac{9}{10})^{\frac{10}{9}}$ (d) 999/1000 (e) 9/10

3. If you lose 20% on an investment during the first year and gain 25% the following year, what is your net gain over the two years?

   (a) 0% (b) 5% (c) 2.5% (d) −5% (e) 1.25%

4. How many divisors does the number **2007** have?

   (a) 2 (b) 3 (c) 4 (d) 6 (e) 8

5. The number 2^{29} is a 9-digit number with distinct digits. Which digit is missing?

   (a) 0 (b) 3 (c) 4 (d) 5 (e) 7

6. If this pattern continues, where would the number 289 apppear?

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                1
            3       5
        7       9       11
    13      15      17      19
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   (a) 8th element in row 16
   (b) 9th element in row 17
   (c) 9th element in row 18
   (d) last element in row 17
   (e) last element in row 18

7. Consider an infinite geometric series with first term a and common ratio r. If the sum is 4 and the second term is $\frac{3}{4}$, then a possible choice of a and r is

   (a) $a = \frac{7}{4}, r = \frac{3}{7}$
   (b) $a = 2, r = \frac{3}{4}$
   (c) $a = \frac{3}{2}, r = \frac{1}{2}$
   (d) $a = 3, r = \frac{1}{4}$
   (e) $a = 1, r = \frac{1}{4}$

8. For all $x \in (0,1)$, which statement is true?

   (a) $e^x < 1 + x$
   (b) $\ln(1+x) < x$
   (c) $x < \sin x$
   (d) $x < \ln x$
   (e) $x + 1 < e^{-x}$

9. A set of 26 encyclopedias (one for each letter) is placed on a bookshelf in alphabetical order from left to right. Each encyclopedia is 2 inches thick including the front and back covers. Each cover (front or back) is 1/4 inch thick. A bookworm eats straight through the encyclopedias, beginning inside the front cover of volume A and ending after eating through the back cover of volume z. How many inches of book did the bookworm eat?

   (a) 48 (b) 48.5 (c) 51.25 (d) 51.5 (e) 51.75

10. What is the smallest positive integer so that $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^n = 1$

    (a) 0 (b) 2 (c) 4 (d) 8 (e) 16

11. One hundred balls labelled 1 through 100 are placed in a bag. Four balls are removed from the bag, one by one. What is the probability that the label on the first ball is higher than the label on the last?

    (a) 5/4 (b) 1/2 (c) 0 (d) 49/50 (e) 4/5

12. What are the dimensions of the rectangle with the largest area that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$?

    (a) 2×3
    (b) $2\sqrt{3} \times 3\sqrt{3}$
    (c) $\sqrt{3} \times \frac{3}{2}\sqrt{3}$
    (d) $2\sqrt{2} \times 3\sqrt{2}$
    (e) $\sqrt{2} \times \frac{3}{2}\sqrt{2}$

13. If you place these expressions in increasing order, which one will be in the middle?

$$(a) \quad \sum_{k=1}^{1000} (-1)^k$$

$$(b) \quad \sum_{k=2}^{20} k^2$$

$$(c) \quad \sum_{k=1}^{\infty} \frac{1}{k}$$

$$(d) \quad \sum_{k=1}^{100} k$$

$$(e) \quad \sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k$$

14. The diagonals of a rhombus are 12 and 24. Determine the radius of the circle inscribed in the rhombus.

(a) $6\sqrt{5}$

(b) $12\sqrt{5}$

(c) $\frac{6}{\sqrt{5}}$

(d) $\frac{12}{\sqrt{5}}$

(e) Cannot inscribe a circle in a rhombus

15. If w, x, y, z are positive real numbers such that $w + x + y + z = 2$, then

$$N = (w + x)(y + z)$$

satisfies

(a) $0 \le N \le 1$

(b) $1 \le N \le 2$

(c) $2 \le N \le 3$

(d) $3 \le N \le 4$

(e) $4 \le N \le 5$

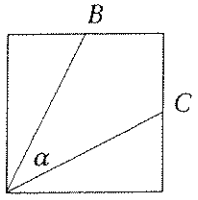
16. As $x \to \infty$, the function $\left(\frac{x-3}{x+2}\right)^x$ approaches

(a) e (b) $\frac{1}{e}$ (c) e^{-5} (d) e^5 (e) 1

17. Triangle ABC has sides 10, 24, and 26 cm long. A rectangle that has an area equal to that of the triangle has width 3 cm. Find the perimeter of the rectangle.

(*a*) 40 cm (*b*) 43 cm (*c*) 56 cm (*d*) 68 cm (*e*) 86 cm

18. Given the square with midpoints B and C. What is the $\sin\alpha$?



(*a*) $\frac{3}{5}$ (*b*) $\frac{4}{5}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{5}}$ (*e*) $\frac{2}{\sqrt{5}}$

19. If the area of a circle is equal to the area of an equilateral triangle, then the ratio of the side of the triangle to the radius of the circle is closest to which number?

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6 (*e*) 7

20. If this multiplication problem works in base b, what is b?

$$(15_b)(15_b) = 321_b$$

(*a*) 4 (*b*) 6 (*c*) 7 (*d*) 8 (*e*) 9

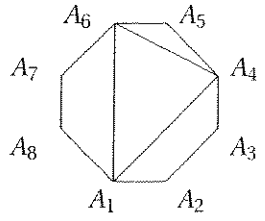
21. A rhombus with sides of 8 cm and an angle of 120^o will have an area closest to.

(*a*) 35 cm^2 (*b*) 45 cm^2 (*c*) 55 cm^2 (*d*) 60 cm^2 (*e*) 65 cm^2

22. If $b > a$, then the equation $(x-a)(x-b)-1=0$ has

(*a*) both roots in $[a, b]$
(*b*) both roots in $(-\infty, a)$
(*c*) both roots in (b, ∞)
(*d*) one root in $(-\infty, a)$ and the other in (b, ∞)
(*e*) one root in $[a, b]$ and the other in (b, ∞)

23. How many different triangles can you draw as in the figure, if the three vertices have to be among the shown points $A_1, \ldots, A_8$?



(a) $8(7)(6)$ (b) 56 (c) $8!$ (d) $3!$ (e) 24

24. What is the value of

$$\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots}}}$$

(a) 1 (b) $\frac{1}{2}$ (c) $1+\sqrt{2}$ (d) $-1\pm\sqrt{2}$ (e) $-1+\sqrt{2}$

25. Paul and Judy play the exciting game "throw a coin six times". If the coin shows heads, Paul gets a point, if tails, Judy gets a point. After six throws, they compare their scores. How likely is it that the game will be a tie?

(a) $\frac{1}{2}$ (b) $\frac{5}{16}$ (c) $\frac{1}{4}$ (d) $\frac{7}{16}$ (e) $\frac{3}{16}$

26. If $f(\sin(x)) = \sin(3x)$, then $f(\cos(30^o)) = ?$

(a) 0 (b) 1 (c) -1 (d) $\sqrt{\frac{3}{2}}$ (e) $\frac{1}{2}$

27. If the equation $\left(\frac{1}{4}\right)^x + \left(\frac{1}{2}\right)^{x-1} + b = 0$ has a positive solution, then the real number b is in what interval?

(a) $-\infty < b < 1$
(b) $-\infty < b < -2$
(c) $-\infty < b < 0$
(d) $-3 < b < 0$
(e) $-\infty < b < -3$

28. If $f(x) = 3x^2 - x + 4$, $f(g(x)) = 3x^4 + 18x^3 + 50x^2 + 69x + 48$, then what is one of the sums of all the coefficients of $g(x)$?

(a) 8 (b) 1 (c) 3 (d) 7 (e) 0

29. Evaluate $\int_1^3 \frac{x^3+x^2+1}{x^2+x}dx$

(a) $4+\ln\frac{2}{3}$

(b) $4-\ln\frac{2}{3}$

(c) $\frac{9}{2}+\ln\frac{3}{4}$

(d) $\frac{9}{2}-\ln\frac{4}{3}$

(e) $\frac{9}{2}-\ln\frac{2}{3}$

30. Find the perpendicular distance of the point $(4,3)$ from the line

$$y=-2x-4.$$

(a) $\sqrt{65}$ (b) 9 (c) $2\sqrt{5}$ (d) $3\sqrt{5}$ (e) 4